National Mathematics Cluster

Nonlinear Dynamics of Natural Systems[+]

Evaluation 2005-2010 and Future Plans

Part A: Cluster organization and development

1. History

Wiktor Eckhaus, Bert Peletier, and Floris Takens can be seen as the 'founding fathers' of the mathematical field of nonlinear systems in the Netherlands. All three are considered within the international community as pioneers of the field and have been leading scientists within their fields of expertise. Moreover, all three have established their own 'schools' that have been (and still are) very influential inside and outside the Netherlands. The majority of the members of the NDNS$^+$ cluster have directly and indirectly been influenced by at least one of these schools. Since the eighties, the research field has developed strongly towards a discipline in which scientific progress is for a large part based on the ability to combine several different points of view. This evolution has initiated many collaboration projects between various (previous) members of the Eckhaus, Peletier, and Takens schools. In this context, the NWO priority program *'Nonlinear Systems'* played an important role, since it offered a first major opportunity for collaboration and interaction between scientists working on nonlinear systems. The formation of the NDNS$^+$ cluster has been the natural next step in the evolution of the field of nonlinear systems in the Netherlands.

The NDNS$^+$ cluster has indeed developed into a strong platform in which the research in the mathematics of nonlinear systems is stimulated and coordinated at a national level. As described in the original proposal, the aim of the cluster has been to foster the exchange of ideas, views, insights, problem formulation and methods amongst scientists interested in mathematics and earth and life sciences. An important objective has been to attract young talented researchers to the field, to motivate them, and to help them get an overview. The cluster has been successful in each of these respects.

2. Leadership and board

The cluster initially consisted of one centre (or hub), Groningen, and three nodes, Leiden, VU Amsterdam, and the CWI. Almost all mathematics departments in the Netherlands had and have members of cluster NDNS$^+$ as employees. During the first four years the board consisted of Henk Broer (Groningen) as the managing director, and of further board members Sjoerd Verduyn Lunel (Leiden), Aad van der Vaart (VU) en Arjen Doelman (CWI). In 2009 the board was extended with Mark Peletier from Eindhoven, that in this way also became a node of the cluster. The board initiates or triggers most cluster activities such as workshops, advisors, the programme for the national Mastermath curriculum, etc., and decides on the corresponding financial consequences.
The board has direct advisory support from NWO, in the persons of Lex Zandee and later Petra de Bont, who also attend the board meetings. The managerial duties in Groningen are supported by the part-time secretary Tini Roek and by the postdoc Konstantinos Efstathiou, who also maintains the NDNS$^+$ website. Financial arrangements of the cluster members are always made directly with NWO.

3. Participating University groups

3.1. Main Universities and their researchers

The NDNS$^+$-cluster has brought together most of the groups working in the field of nonlinear mathematics. Below we list only the permanent members.

- Center for Mathematics and Computer Science (CWI) (node): Blom, Crommelin, Doelman (until 2009), Ebert, Frank, Hundsdorfer, Klau, Koren, Merks, Rademacher, Sommeijer, Van Schuppen, Verwer
- University of Groningen (RuG) (hub): Broer, Çamlibel, Van Enter, Van den Heuvel, Van der Schaft, Vegter, Verbitskiy, Waalkens, Wit
- Delft University of Technology (TUD): Heemink, Van Neerven, Vuik

- Eindhoven University of Technology (TU/e) (node since 2009): Anthonissen, Van Brummelen, Molenaar (until 2008), Muntean, Peletier, Pop, Prokert, Rienstra
- University of Amsterdam (UvA): Brandts, Hek (until 2008), Homburg, Koornwinder, Wiegerinck, Stevenson, Stolk
- University Leiden (UL) (node): Doelman (starting 2010), Van Gaans, Haccou, Hille, Den Hollander, De Jeu, Meulman, Redig, Rottschäfer, Verduyn Lunel
- University Twente (UT): Bokhove, Van Gils, Zagaris
- Utrecht University (UU): Diekmann, Dijkstra, Kuznetsov, Zegeling
- VU University Amsterdam (VUA) (node): Van den Berg, Bijma, De Gunst, Hulshof, Jongbloed (until 2007), Jonker, Planqué, Rink, Van der Vaart, Van der Vorst

3.2. Cooperation

In the NDNS$^+$ cluster, the research area of nonlinear systems – or, in a somewhat broader definition, applied analysis – is represented by way of active research groups at nearly all major universities in the Netherlands. In the original set-up of the cluster, four of these groups – CWI, VU Amsterdam, Groningen, and Leiden – had a special position as hub (Groningen) or as node (the other three). This was a natural consequence of the application procedure for the (initiation of the) NDNS$^+$ cluster: according to the original guidelines, a cluster could only have one hub and a very limited number of nodes (the three nodes of NDNS$^+$ formed a maximal possible number). This had consequences for the organization of the cluster, since – once again, according to the guidelines – a cluster was only allowed to fund research positions at the hub or at a node. Of course this created – certainly from the scientific point of view – an artificial distinction between these four central groups and the other four groups (at Twente, Utrecht, Amsterdam, and Eindhoven): as was originally intended, the main impact of the cluster formation has been the creation of new permanent positions in the hub and initial nodes (see also section 6). The groups at Twente, Utrecht, Amsterdam, and Eindhoven have not been able, or in fact allowed, to take part in this.

Nevertheless, in the course of the years, the NDNS$^+$ cluster has succeeded in setting up an organizational structure that reduced the impact of this artificial distinction. In fact, the cluster has actively worked on this. In early 2009, it became clear that most of the budget that was originally planned for long-term visitors had not been used (for very good reasons: it has been NDNS$^+$ policy to request that an application for support for a long time visitor also be submitted to an alternative funding agency – these other parallel proposals turned out to be extremely successful). It was decided that this money should be invested in a call for proposals for pre-financing of new permanent positions and/or postdocs. It was also decided that the hub and nodes would take a natural step back during this procedure. This had a very beneficial outcome: NDNS$^+$ had the opportunity to pre-finance two tenure track positions, one at Twente (Zagaris) and one at the University of Amsterdam (Peters), that would not have existed without the cluster; moreover, NDNS$^+$ was also able to fund a postdoc position in Eindhoven (that will be extended by Eindhoven University of Technology). This illustrates that the NDNS$^+$ cluster intended and succeeded to grow beyond the artificial distinction that appeared as an unintended side-effect of the original application procedure.

In the present situation, the cluster has evolved into a genuine cooperation between all 'nonlinear mathematicians' in the Netherlands – exactly as was originally intended. Of course, the field of nonlinear dynamical systems/applied analysis is huge, so within this field there are several specializations in which only a limited number of groups participate (for instance, there are strong connections between the groups at Eindhoven and at the VU Amsterdam with a focus on variational methods, etc.). This is a natural, and from the scientific point of view, desirable situation. The success of the recently organized 'birthday celebration' – the workshop of April 2010 in Eindhoven that celebrated the fifth anniversary of the NDNS$^+$ cluster – has shown that the cluster has been able to play the role it always has aspired to.

However, after the original application procedure, the cluster has been expanded: it has become more than the above described national research team for 'nonlinear mathematicians', the '+' in the name of the cluster indicates that there is more to the cluster than the `Nonlinear Dynamics of Natural Systems'. The '+' refers to the additional dimension that spans activities in stochastics, with a strong emphasis on statistics and especially bio-statistics. The motivation for adding this '+' is scientifically obvious: a significant interaction between mathematics and life sciences – one of the major themes of $NDNS^+$ – necessarily includes a (bio-) statistical aspect. In our opinion, adding the '+' has worked very well: within $NDNS^+$ many activities have been organized in which the forces of both groups of mathematicians have been combined (see sections 6, 7, and 8). In the interactions with life scientists it is clear that mathematicians should join forces, and that is exactly what has happened. Nevertheless, the situation changed with the recent formation of the STAR cluster, in which all mathematical research in the fields of probability and stochastics is concentrated. Especially since both $NDNS^+$ and STAR look forward to joining forces in our interactions with the life sciences, it is no longer sensible to maintain the special bio-statistical dimension of the $NDNS^+$ cluster. In our opinion, clusters should not act as completely independent entities: therefore we consider the fact that bio-statistics has been an integral part of the cluster for five years as very positive: it gives a jump-start to our cooperation with the STAR cluster.

The cluster will continue to organize activities, and will continue to operate under the name 'Nonlinear Dynamics of Natural Systems', including the stochastic '+'. The research field is rapidly evolving, and so is the academic landscape, and therefore certain changes will be made. These are outlined in Part C of this document.

4. Expenditure and funding

4.1. Funded research positions and advisors

RuG:
- Dr. M.K. Çamlibel, tenure track, 1 fte, 4 years
- Prof. dr. E.R. van den Heuvel (from Schering-Plough), professor, 0.2 fte, 2 years
- Prof. dr. A.J. van der Schaft, professor, 0.9-1 fte, 5 years
- Prof. dr. E. Verbitskiy (from Philips), professor, 0.2 fte, 2 years
- Prof. dr. H. Waalkens, professor, 1 fte, 4 years

CWI:
- Dr. D.T. Crommelin, tenure track, 1 fte, 4 years
- Dr. J. Rademacher, tenure track, 1 fte, 3 years
- Dr. A. Zagaris, postdoc, 1 fte, 1 year
- Drs. J. Zijlstra-Jichkina, 1 fte, 1 year (and 3 years CWI)
- Dr. M. Chirilus-Bruckner, postdoc, 1fte, 1 year

UL:
- Dr. S.C. Hille, tenure track, 1 fte, 4 years
- Prof. dr. J.J. Meulman, professor, 0.2 fte, 4 years
- Dr. V. Rottschäfer, tenure track, 1 fte, 3 years

TU/e:
- Dr. S. Arnrich, postdoc, 1 fte, 2 years (and 1 year TU/e)

UT:
- Dr. A. Zagaris, tenure track, 1 fte, 1 year (and 4 months CWI)

UvA:
- Dr. H. Peters, tenure track, 0,5 fte, 2 years

VUA:
- Dr. F. Bijma, lecturer, 0,5 fte, 5 years

- Prof. dr. M.C.M. de Gunst, professor, 0.4 fte, 4 years
- Dr. M. Jonker, lecturer, 0.8 fte, 5 years
- Dr. R. Planqué, tenure track, 0.7 fte, 5 years

4.2. Advisors and visitors

Advisors

- S.A. van Gils, professor, from UT to UU, 3 years one day a week
- H. Hanßmann, lecturer, from UU to RuG, 1 year one day a week
- G. Hek G, lecturer, from UvA to CWI, 1 year one day a week
- S. Pop, lecturer, from TU/e to UU, 2 years one day a week
- G. Prokert, lecturer, from TU/e to VUA, 4 years one day a week
- F. Redig, lecturer, from UL to RuG, 2 years one day a week
- V. Rottschäfer, lecturer, from UL to CWI, 3 years one day a week
- H.E. de Swart, professor, from UU to CWI, 4 years one day a week
- A. van der Vaart, professor, from VUA to UL, 4 years one day a week

Long-term visitors (two months or more)

- V. Sidoravicius,* Instituto Nacional de Matemática Pura e Aplicada, Brazil, 3 months, 2005
- S. Angenent,* University of Wisconsin, USA, 12 months, 2005-2006
- B. Davidovitch,* University of Massachusetts, USA, six months, 2006-2007
- W.D. Kalies, Florida Atlantic University, USA, two months, 2006
- D. Siegmund, Stanford University, USA, four months, 2006
- G. Derks, University of Surrey, UK, three months, 2007
- T. Kaper,* Boston University, USA, three months, 2007
- M. Krupa,* New Mexico State University, USA, six months, 2007
- R. Fernandez, Université de Rouen, France, eighteen months, 2007-2008
- S. van Strien, University of Warwick, UK, eleven months, 2008-2009
- K.-I.Ueda, Kyoto University, Japan, two months, 2009
- W. Jäger, Universität Heidelberg, Germany, four months, 2009-2010 (KNAW Akademie-professor)
- K. Promislow, Michigan State Universtiy, USA, two months, 2010 (Kloosterman professor)

*pre-financed by NDNS+, later paid by other funding agencies

In addition a large number of short-term visits were financed with NDNS+ funds.

The success of the advisor and visitor programmes can be illustrated in various ways. High-profile visitors, such as Angenent, Mimura, and Jäger have been brought to the Netherlands, and their presence has benefited many more scientists (also outside of mathematics) than those of the cluster alone. The pre-financing done by the cluster has been very effective in allowing the organisers of the visit to press forward with the practical arrangements while applying for additional funding elsewhere. This leads to a significant speedup of the life cycle of a visit.

The advisor programme has made several co-supervised PhD projects possible (Zaal, Nolet, Zijlstra, Valkhoff), has given rise to a large number of joint publications (see the publication list) and has thus contributed to the mathematical infrastructure of the Netherlands.

4.3. Expenditure of the remaining cluster money

The remaining funds contributed to a variety of activities. Two classes of expenditure are worth separate mention. The first is the funding of students of the MRI master class (MRI Stieltjes 2005-6 € 40.597, and MRI 2009-10 € 20.000). Involvement with education became a priority in the course of the last five years, and will become even more so in the future plans (see Part C below). In the coming years the cluster will become more directly involved with education at PhD and Master level, and will organize separate Summer and Winter Schools and other events for this purpose.

The second class of expenses is the support of workshops. We organized 9 special NDNS+ workshops (in total € 100.000; see section 5.2), and the cluster financially supported 36 'related' workshops and conferences (in total € 80.000). This class of activities is very fruitful, and will be intensified in the future (again, see Part C).

4.4. Added funding by universities, industry, and research institutes

Here we list the financial contributions to NDNS$^+$activities by various organizations. Grants are mentioned separately under 5.3 below.

As compensation for the managing directorship of H.W. Broer and in support of the cluster:
- K. Efstathiou (RuG) postdoc, 1 fte, 5 years (€ 300.000,-)
- T.G. Roek (RuG), executive secretary, 0,5 fte, 5 years (€ 150.000,-)

The CWI has supported the cluster through:
- J. Zijlstra-Jinchkina (CWI), PhD student, 3 years (€ 135.000)
- A. Zagaris (CWI), postdoc, 4 months (€ 20.000)

The postdoc position of Martina Chirilus Bruckner was supplemented with € 13.800 by the Universität Karlsruhe and with € 2.550 by CWI.

TU/e will support the cluster (in 2011) through:
- S. Arnrich (TU/e), postdoc, 1 year (€ 60.000)

The following Vernieuwingsimpuls grants are matched by the respective universities:
- M.A. Peletier (VIDI 2003, € 200.000 from TU/e)
- F. Pasquotto (VENI 2006, € 70.000 from VUA)
- B. Rink (VENI 2007, € 70.000 from VUA)
- M. Bootsma (VENI 2007, € 70.000 from UU)
- V. Rottschäfer (VIDI 2006, matching of € 200.000 and a postdoc (€ 120.000) from UL)
- O. van Gaans (VIDI 2005, € 200.000 from UL)
- A. Zagaris (VENI 2006, € 70.000 from UvA)

Note that significant, long-term additional financial commitments are represented by the permanent positions that the tenure-track appointments lead into. These are Çamlibel, Van der Schaft, and Waalkens (RuG), Crommelin and Rademacher (CWI), Hille and Rottschäfer (UL), Zagaris (UT), Peters (UvA), and Jonker and Planqué (VUA).

Several projects of the cluster are partly or completely funded by industry:
- Five STW-projects include matching from industry, contributed by Oranjewoud BV (€ 450.000), Philips Lighting ((€ 340.000), Liandon (€ 100.000), National Space Institute Denmark (€ 94.000), Circlair BV (€ 70.000), KEMA (€ 52.000), NLR (E 50.000), Siemens (€ 32.000), van der Heide Group (€ 28.000), and Bradford Engineering BV (€ 24.000)
- Two research projects are completely funded by industrial partners: ABB Corporate Research (€ 390.000) and Tejin Aramid (€ 280.000)

5. Performance

5.1. Doctoral degrees

2005:

- Bijma, F. (VUA): *Mathematical modelling of magnetoencephalographic data*, supervisor: R.M. Heethaar, co-supervisor: J.C. de Munck (July)
- Gulikers, J. (RuG): *Reinvention of geometry,* supervisor: H.W. Broer; co-supervisors: J.A. van Maanen and A. van Streun (December)
- Huu Khanh, N. (UvA): *Heteroclinic Cycles in Thermal Convection Models,* supervisor: A. Doelman, co-supervisor: A.J. Homburg (December)
- Meulen, F. van der (VUA): *Statistical estimation for Lévy-driven OU processes and Brownian semimartingales*, supervisor: A.W. van der Vaart; co-supervisor G. Jongbloed (June)
- Montijn, C. (TU/e): *Evolution of Negative Streamers in Nitrogen: a Numerical Investigation on Adaptive Grids*, supervisors: U. Ebert and W. Hundsdorfer (December)
- Ploeg, H. van der (UvA): *Singular Pulse Patterns in the Gierer-Meinhardt Equation* Promotor: A. Doelman (June)
- Saleh, K. (RuG): *Organising centers in the semi-global analysis of dynamical systems*, supervisor: H.W. Broer (December)
- Terra, G. (UvA): *Nonlinear Tidal Resonance*, supervisor: A. Doelman, co-supervisor: L. Maas (October)

2006:

- Jong, K. (VUA): *Machine Learning for Human cancer research,* supervisors: A.W. van der Vaart, G. Eiben, co-supervisor: E. Marchior (June)
- Meulenbroek, B. (TU/e): *Streamer branching: conformal mapping and regularization*, supervisors: U. Ebert and J. Hulshof (April)
- Pham Thi, N.N. (UvA): *Numerical Analysis of Phytoplankton Dynamics*, supervisors: J.G. Verwer and J. Huisman (November)
- Trapman, P. (VUA): *On stochastic models for the spread of infections,* supervisors: R.W.J. Meester and J.A.P. Heesterbeek (UU) (September)
- Valkhoff, N. (UvA): *Stabilization by Competing Instability Mechanism,* supervisor A. Doelman, co-supervisor: G.M. Hek (November)

2007:

- Boldin, B. (UU): *Mathematical aspects of infectious disease dynamics*, Reinhart Heinrich Doctoral Thesis Award 2007, supervisors: O. Diekmann and M.J.M. Bonten (September)
- Briels, T.M.P. (TU/e): *Exploring streamer variability in experiments,* supervisors: U. Ebert, G.M.W. Kroesen, and E.M. van Veldhuizen (December)
- Gillett, A.J. (VUA): *Phase transitions in Bak-Sneppen avalanches and in a continuum percolation model*, supervisor: R.W.J. Meester (Month?)
- Guyonne, V. (VUA): *Mathematical models for flame balls*, supervisors: J. Hulshof and C.M. Brauner, co-supervisor: G.J.B. van den Berg (September)
- Horst, S. ter (VUA): *Relaxed commutant lifting and Nehari interpolation*, supervisors: M.A. Kaashoek and A.C.M. Ran (November)
- Kooij, A. van der (UL, Cum Laude): *Prediction accuracy and stability of regression with optimal scaling transformations*, received the 2007 Distinguished Dissertation Award of the Classification Society of North America, supervisor: J.J. Meulman (June)
- Linting, M. (UL): *Nonparametric inference in nonlinear principal components analysis. Exploration and beyond*, supervisor: J.J. Meulman (October)
- Setiawan, A. (VUA): *Statistical analysis of genetic data in twin studies and association studies*, supervisor: A.W. van der Vaart (April)
- Sitters, M.H. (RuG): *Sybrandt Hansz Cardinael 1578-1647, Rekenmeester en wiskundige, zijn leven en zijn werk*, supervisors: H.W. Broer and J.A. van Maanen (UU) (November)

- Welten, M.C.M. (UL): *Spatio-temporal gene expression analysis from 3D in situ hybridization images*, supervisors: S.M. Verduyn Lunel and H.P. Spaink, co-supervisor: F.J. Verbeek (November)
- Zareba, P.M. (VUA): *Representations of Gaussian processes with stationary increments*, supervisor: A.W. van der Vaart, co-supervisors: J.H. van Zanten and K.O. Dzhaparidze (December)

2008:

- Asadi, E. (VUA): *Integrable systems in symplectic geometry,* supervisor: J. Hulshof, co-supervisors: J.A. Sanders and J.P. Wang (May)
- Chandramouli, S. (RuG): *Renormalization and non-rigidity,* supervisors: H.W. Broer and M. Martens (December)
- Elrofai, H. (VUA): *Stability of radiative propgating flames*, supervisors: J. Hulshof and J.B. van den Berg (May)
- Fey-den Boer, A.C. (VUA): *Sandpile models: The infinite volume model, Zhang's model and limiting shapes*, supervisor: R.W.J. Meester, co-supervisor: F. Redig (March)
- Gennip, Y. van (TU/e): *Partial localisation in a variational model for diblock copolymer-homopolymer blends*, supervisor: M.A. Peletier (October)
- Hazard, P. (RuG): *Hénon-like maps and renormalization*, supervisors: H.W. Broer and M. Martens (December)
- Hupkes, H.J. (UL): *Invariant manifolds and applications for functional differential equations of mixed type*, supervisor: S.M. Verduyn Lunel, Received Rubicon-grant for a two year visit at the Brown University (Prof. Sandstede) (June)
- Kruijer, W. (VUA): *Convergence rates in nonparametric Bayesian density estimation*, supervisor: A.W. van der Vaart (June)
- Lukina, O.V. (RuG): *Geometry of torus bundles in integrable Hamiltonian systems,* supervisor: H.W. Broer (September)
- Savcenco, V. (UvA): *Multirate numerical integration for ordinary differential equations*, supervisor: J.G. Verwer, co-supervisor: W. Hundsdorfer (January)
- Subramanian, E.N. (RuG): *Attractor switching in neuron networks and spatiotemporal filters for motion processing*, supervisors: H.W. Broer and N. Petkov (February)
- Wójcik, W.T. (VUA): *Braids, floer homology and forcing in two and three dimensional dynamics*, supervisors: R.C.A.M. van der Vorst and J.B. van den Berg (March)
- Zmarrou, H. (UvA): *Bifurcation of random maps,* supervisor: A. Doelman, co-supervisor: A.J. Homburg (March)

2009:

- Ashyraliyev, M. (UvA): *Modelling, simulation, and inferring regulatory networks,* supervisor: J.G. Verwer (October)
- Banachewicz, K. (VUA): *A collection of problems in credit risk modelling*, supervisor: A.W. van der Vaart, co-supervisor: A. Lucas (June)
- Dam, A. van (UU): *Moving mesh methods for compressible flow problems,* supervisors: H.A. van der Vorst and P.A. Zegeling (July)
- Driesse, R. (UvA): *Bifurcations from robust homoclinic cycles*, supervisor: A. Doelman, co-supervisor: A.J. Homburg (November)
- Heijster, P. van (UvA): *Front interactions in a three-component system,* supervisor: A. Doelman (May)
- Hlod, A. (TU/e): *Curved jets of viscous fluid: Interactions with a moving wall,* supervisor: M.A. Peletier (September)
- Holtman, S.-J. (RuG): *Dynamics and geometry of resonant bifurcations,* supervisors: H.W. Broer and G. Vegter (September)
- Kramar, K. (VUA): *Conley index theory for braids and forcing in fourth order conservative systems*, supervisors: R.C. van der Vorst and J.B. van den Berg (February)
- Li, C. (CWI): *Joining particle and fluid aspects in streamer simulations,* supervisors: U. Ebert and W. Hundsdorfer (February)

- Nemcova, J. (VU): *Rational systems in control and system theory,* supervisor: J.H. van Schuppen (February)
- Opoku, O. (RuG): On *Gibbs properties of transforms of lattice and mean-field system,* supervisors: Ch. Külske, A.C.D. van Enter and H.W. Broer (September)
- Panhuis, P. in 't (TU/e): *Mathematical aspects of thermoacoustics,* supervisors: S.W. Rienstra, J. Molenaar and J.J.M. Slot (June)
- Vondenhoff, E. (TU/e): *Hele-Shaw and Stokes flow with a source or sink: Stability of spherical solutions*, supervisor: M.A. Peletier (June)

2010:

- Ashyraliyev, M. (UvA): *Modelling, simulation, and inferring regulatory networks,* supervisor: J. G. Verwer (October)
- Härdin, H.M. (VU): *Handling biologic complexity: as simple as possible, but not simpler,* supervisor: H.V. Westerhoff and J.H. van Schuppen; co-supervisors: K. Krab and A. Zagaris (June)
- Hazewinkel, J. (UU): *Attractors in stratified fluids,* supervisors: L. Maas, A. Doelman, co-supervisor: S. Dalziel (Cambridge) (March)

5.2. Seminars, workshops and conferences

Nine workshops were organized as 'pure' NDNS+-workshops:

- *Mathematics of life sciences*, Kick-off workshop of NDNS+, October 10 - 13 2005, RU Groningen, organisers: HW Broer, A Doelman, A.W. van der Vaart and S.M. Verduyn Lunel
- *Dynamics of nonlinear waves*, April 24-28 2006, RU Groningen, organisers: H.W. Broer, A. Doelman et al
- *Mathematics of earth sciences*, November 28-December 1 2006, RU Groningen, organisers: H.W. Broer, H.A. Dijkstra (IMAU), A. Doelman and H.E. de Swart (IMAU)
- *Mathematical modeling and analysis of biological networks*, January 29 – February 2 2007, LC, Leiden, organisers: A.W. van der Vaart and S.M. Verduyn Lunel
- *Control theory for systems biology*, November 12-14 2007, RU Groningen, organisers: A.J. van der Schaft and J.H.van Schuppen
- *Partial differential equations in applied analysis*, May 22-23, 2008, Apeldoorn, organiser: G. Prokert
- *Hitting, returning and matching in dynamical systems, information theory and mathematical biology*, November 3 - 7 2008, Eurandom, Eindhoven, organisers: F. den Hollander, F. Redig et al
- *KAM Theory and its applications*, December 1-5 2008, LC, Leiden, organisers: H.W. Broer, H. Hanßmann et al
- *Statistics and the life sciences: High-dimensional inference and complex data*, November 23-25 2009, RU, Groningen, organisers: E.C. Wit, E. van den Heuvel and S. Heisterkamp
- *Nonlinear dynamics of natural systems*, April 13-16 2010, Eurandom, Eindhoven, organisers: H.W. Broer, A. Doelman, A. Muntean, M.A. Peletier, A.W. van der Vaart and S.M. Verduyn Lunel

In addition, a total of **36** other workshops were supported by NDNS+, with amounts varying from € 1000 to € 5000.

5.3. Honored research proposals

2005:

- Bokhove, O. (UT) and J. Frank (CWI), *Hamiltonian-based numerical methods in forced-dissipative climate predictions*, 2005, 2006-2011, NWO, € 170.000
- Broer, H.W. (RuG) and H.A. Dijkstra (IMAU), *Atmospheric variability and the atlantic multidecadal oscillation*, 2005, 2006-2010, NWO ALW, € 187.214
- Doelman, A. (CWI), L. Maas (NIOZ), *Internal wave patterns in 3D*, 2005, 2006-2010, NWO, € 199.000

- Ebert, U. (CWI), W. Hundsdorfer (CWI), E.M. van Veldhuizen (TU/e), M.D. Bowden (TU/e), G.M.W. Kroesen (TU/e), E.J.M. van Heesch (TU/e), *Electric "fracture": growth and branching of ionized channels*, 2005, 2005-2009, STW, 1 PhDs, 5 years PD plus 210 k€ budget
- Ebert, U. (CWI), *Moving ionization boundaries and charge transport in early stages of sparks and lightning*, 2005, 2005-2009, NWO, € 171.000
- Gaans, O. van (UL), *Stationary dynamics in infinite dimensions*, 2005, 2006-2010, NWO-VIDI, € 600.000
- Hollander, F. den (UL), *Random copolymers in random emulsions*, 2005, 2005–2006, NWO, € 120.000
- Meulman, J.J. (UL), *Nonlinear categorical data analysis*, SPSS, 2005, 2005-2009, E 2.057.763
- Prokert, G. (TU/e), *Stability of moving boundaries in Hele-Shaw and Stokes flows*, 2005, 2005-2009, Open Competitie, € 200.000
- Rienstra, S.W. (TU/e), *Turbomachinery noise radiation through the engine exhaust*, 2005, 2005-2007, EC (FP6/Aeronautics and Space), € 173.000

2006:
- Broer, H.W. (RuG) and H. Nijmeijer (TU/e), *Stability, bifurcations and stabilisation of invariant sets in defferential inclusions*, 2006, 2009-2013, NWO EW, RuG, € 172.000
- Frank, J. (CWI), *Adaptive multisymplectic box schemes for Hamiltonian wave equations*, 2006, 2007-2011, NWO, € 172.000
- Hollander, F. den (UL), *Wulff shape of critical droplets under stochastic dynamics*, 2006, 2006–2007, NWO, € 120.000
- Hundsdorfer , W. (CWI), *Monotonicity preservation for general multistep methods*, 2006, 2007-2011, NWO, € 172.000
- Pasquotto, F. (VUA), *Symplectic structures: at the interface of analysis, geometry and topology*, 2006, 2007-2010, NWO-vernieuwingsimpuls VENI, € 200.000
- Rottschäfer, V. (UL), *Formation of singularities in natural systems*, 2006, 2007-2011, NWO-VIDI and ASPASIA, € 600.000 + € 100000
- Vaart, A.W. van der (VUA), *Analysis of MEG signals*, 2006, 2006-2010, VU Excellence program, € 180.000
- Zagaris, A. (CWI/UvA), *Reduced Models for multiscale reaction-diffusion dynamics*, 2006, 2006-2009, NWO-vernieuwingsimpuls VENI, € 208.000

2007:
- Berg, J.B. van den (VUA), *Studiegroep Wiskunde met de Industrie*, 2007, 2008-2012, EW-NWO/STW, € 150.000 with G. Hek, M.A. Peletier, V. Rottschäfer, R. van der Hofstad
- Bootsma, M. (UU), *Mathematical tools in support of infection prevention*, 2007, 2007-2010, NWO VENI, € 200.000
- Pemen, A.P.M. and U. Ebert (CWI), *Power modulation and corona-plasma for environmental purposes*, 2007, 2007-2010, SenterNovem, € 350.000
- Hupkes, H.J. (UL/Brown University), *Understanding Waves and Patterns in Discrete Media: Towards a Geometric Approach*, 2007, 2008-2010, NWO Rubicon, € 80.000
- Külske, C. (RuG) and A.C.D. van Enter (RuG), *Interacting stochastic models on small-world net-works,* 2007, 2008–2012, NWO EW, € 180.371
- Pop, I.S. (TU/e), *Non-linearities and upscaling in porous media*, 2007, 2007-2011, International Research Training Group, NWO/DFG, € 825.000, involves 4 universities in Germany and the Netherlands
- Rink, B. (VUA), *Hamiltonian lattice dynamical systems*, 2007, 2007-2011, NWO-vernieuwingsimpuls VENI, € 208.000
- Schaft, A.J. van der (RuG), *Compositional Analysis and Control of Hybrid Systems*, 2007, 2007-2011, NWO EW Open Competitie, € 172.000

2008:
- Ebert, U. (CWI), W. Hundsdorfer (CWI), J. van Dijk (TU/e), E.M. van Veldhuizen (TU/e), M. Haverlag (Philips Lighting), *The start up of lighting and lightning: Streamer discharges in lamp*

ignition, electric switches and materials processing, 2008-2013, STW, 2 PhDs, 3 years PD plus 36 k€ budget

- Gils, S.A. van (UT), *From spiking neurons to brain waves, NWO Computational Life-Sciences II*, 2008, 2008-2012, € 200.000
- Gunst, M.C.M. (VUA), *Neuronal network formation through reciprocal interactions between activity and structure*, 2008, 2008-2011, NWO-CLS, € 460.000, with A. van Ooyen and J. van Pelt
- Jeu, M. de (UL), *Banach algebra dynamical systems and positivity*, 2008, 2009-2013, NWO, € 182.495
- Meulman, J.J. (UL), *PASCAL2* (European Network of Excellence), 2008, 2008-2009, € 5.500
- Pop, I.S. (TU/e), *Second generation of integrated batteries*, 2008, 2008-2012, STW, € 500.000
- Rink, B. (VUA), *Variational methods for quasi-periodicity*, 2008, 2009-2012, NWO Vrije Competitie, € 186.495
- Schaft, A.J. van der (RuG) and J.M.A. Scherpen (RuG), *Structure preserving model reduction for port Hamiltonian systems*, 2008, 2009-2013, NWO EW Vrije Competitie, € 190.000 (each applicant 50%)
- Slot, J.J.M. (TU/e), *Constitutive modelling of concentrated solutions of main-chain LCP's*, 2008, 2008-2012, Tejin Aramid BV, € 280.000
- Vaart, A.W. van der (VUA), *Statistics for high-dimensional data concerning molecular markers for cancer progression,* 2008, 2008-2012, VU-IBIVU, € 220.000 with M. van der Wiel
- Vaart, A.W. van der (VUA), *Statistical Genetics*, NWO Center for Medical Systems Biology, 2008, 2008-2012, € 200.000.
- Vaart, A.W. van der (VUA), *Statistics for high-dimensional data concerning molecular markers for cancer progression*, NWO Center for Medical Systems Biology, 2008, 2008-2012, € 200.000.

2009:

- Chirilus-Bruckner, M. (CWI), A. Doelman (CWI), J. Rademacher (CWI), *Semi-starke interaction lokalisierter Strukturen in musterbildenen Systemen,* 2009, 2009-2010, Karlsruhe Institute of Technology
- Crommelin, D. (CWI), F. Selten (KNMI), P. Siebesma (KNMI/TU Delft), H. Jonker (TU Delft), *Influence of a new stochastic convection parameterization on cloud-climate feedbacks*, 2009, 2010-2014, NWO ALW (Feedbacks in the Climate System), € 214.000
- Donne, A.J.H. (FOM-Rijnhuizen), M.R. de Baar, R. Jaspers (FOM-Rijnhuizen), E. Westerhof (FOM-Rijnhuizen), U. Ebert (CWI), B. Koren (CWI), M. Steinbuch (TU/e), N.J. Lopes Cardozo (TU/e), *Active Control of Magneto-hydrodynamic modes in Burning Plasmas*, 2009, 2010-2014, FOM, 2 PhDs for CWI (15 PhDs total, program budget € 3.400.000)
- Gunst, M.C.M. de (VUA), *Applied coagulation systems biology to accurately assess the hemostatic balance of individual patients,* 2009, 2009-2013, STW, € 2.264.565 with P.H. Reitsma, Ph.G. de Groot, C.A.J. Klaassen
- Gunst, M.C.M. de (VUA), *Transcriptional network modeling*, 2009, 2010-2011, Neuroscience Campus Amsterdam, € 62.000
- Haccou, P. (CML, UL), S.C. Hille (MI, UL), M. Emmerich (LIACS, UL), O. Kuipers (RuG), *The evolution of stochastic heterogeneous networks as bet-hedging adaptations to fluctuating environments (BetNet)*, 2009, € 460000
- Heijster, P. van (CWI), *Interaction in a three-component reaction-diffusion model in two spatial dimensions*, 2009, 2009-2011, NWO Rubicon, € 48.000
- Hofstad, R. van der (TU/e) and F. den Hollander (UL), *Random walks on high-dimensional incipient infinite clusters*, 2009, 2009–2010, NWO, € 120.000
- Meulman, J.J. (UL), *Megavariate diagnostics tools for system-based interventions*, LACDR, 2009, 2009-2010, € 111.000
- Peletier, M.A. (TU/e), *Singular limit analysis of metapatterns*, 2009, 2009-2013, NWO Vrije Competitie, € 400.000
- Peletier, M.A. (TU/e), *Fronts and interfaces in science and technology*, 2009, 2009-2012, EU FP7 Marie-Curie Initial Training Network, € 4.000.000 with 14 partners

- Schaft, A.J. van der (RuG), J.M.A. Scherpen (RuG), S. Stramigioli (UT), *Energy-efficient design and control of mobile robotic sensor networks,* 2009, 2010-2014, STW (ASSYS), € 400.000
- Schaft, A.J. van der (RuG), *Systems biology centre for energy metabolism and ageing* (SBC-EMA), 2009, 2010-2014, NWO en ZonMw systeembiologie, € 4.000.000 with 15 partners
- Vaart, A.W. van der (VUA), *Statistics for very high dimensional semi parametric models*, 2009, 2010-2013, NWO Vrije Competitie, € 400.000

2010:

- Bijma, F. (VUA), *Statistics for neuronal network formation*, 2010, 2011-2012, Neuroscience Campus Amsterdam, € 62.000
- Deursen, A.P.J. van (TU/e), U. Ebert (CWI), *Understanding lightning: From terrestrial Gamma-Ray flashes to lightning protection*, 2010, 2010-2014, STW, € 530.000
- Heesch, E.J.M. van (TU/e), U. Ebert (CWI), E.M. van Veldhuizen (TU/e), *Exploring a new medium for high-power switching: Supercritical fluids,* 2010, 2010-2014, STW, € 634.000
- Pasquotto, F. (VUA), *Invariants and dynamics in symplectic geometry*, 2010, 2010-2014, NWO Meervoud, € 210.000
- Pemen, A.J.M. (TU/e), U. Ebert (CWI), E.M. van Veldhuizen (TU/e), *Transient plasma for air purification*, 2010, 2010-2014, STW, € 720.000

6. Added value of cluster formation

The impact of the cluster can be recognized in at least two distinct forms: on one hand in new positions, and on the other hand in new collaborations.

The formation of the NDNS$^+$ cluster has created a significant number of permanent positions (mostly) for young researchers in the field. In the hub in Groningen, three fulltime positions have been funded by the NDNS$^+$ cluster, and a total of seven fulltime positions in the three initial nodes – CWI, Leiden, VU Amsterdam (see section 4.1). Together with the two tenure-track positions at Twente University and the University of Amsterdam, for which the pre-funding through the cluster has been crucial, a total of twelve new fulltime positions have been created in the context of the NDNS$^+$ cluster. Given the relative small scale of mathematics in the Netherlands, this indeed has had a major (positive!) impact on Dutch mathematics.

The organization of workshops has been another central theme within the cluster. Traditionally, there has already been a stimulating exchange of ideas between Dutch nonlinear mathematicians. However, before the formation of the cluster, the contacts between these mathematicians and, for example, biologists or meteorologists, have mostly been restricted to individual, or local, cooperation projects. Especially through the organization of workshops – for instance with topics centered around the interactions between mathematics and life sciences or earth sciences – the interactions between mathematicians and related scientists has become much less restricted, and thus much richer. For instance in the case of earth sciences, through the cluster a structural interaction has been formed between mathematicians and earth scientists at the IMAU (Dijkstra, de Ruijter, de Swart) and the NIOZ (Gerkema, Maas, Ridderinkhof) that transcends the individual level. A similar development has taken place in the life sciences, for instance in the context of systems biology.

As a direct result of the cluster formation, a new human infrastructure has been formed, a network of mathematicians and other scientists revolving around a common interest in problems from natural systems.

7. Knowledge transfer

7.1. Cooperation with other disciplines, joint publications, and joint events

There are too many examples of interdisciplinary collaboration in NDNS$^+$ for a full list here. Instead we describe some examples, one from ecology, one from biophysics, some from biology, and one from medicine. Cluster participants are indicated in bold face.

Colonies of social insects are able to solve complex problems with simple means, as long as those means are used collectively. Through an interplay between mathematicians at the VU University Amsterdam and experimental biologists and computer scientists at Bristol University (the 'Bristol Ant Lab'), we are getting a better understanding of how collective decisions in such colonies are constructed, directed, and implemented.

- Sendova-Franks, A.B., K. Rebecca, B. Hayward, B. Wulf, T. Klimek, R. James, **R. Planqué**, N.F. Britton and N.R. Franks, *Emergency networking: famine relief in ant colonies*, Anim. Behav. doi:10.1016/j.anbehav.2009.11.035, 2009
- Marshall, J.A.R., R. Bogacz, A. Dornhaus, **R. Planqué**, T. Kovacs, N. Franks, *On optimal decision making in brains and social insect colonies*, Proc. Roy. Soc. Interface **6**, pp. 1065-1074, 2009
- **Planqué, R.,** F.X. Dechaume-Moncharmont, N. Franks, T. Kovacs, J. Marshall, *Why do house-hunting ants recruit in both directions,* Naturwissenschaften **94** (11), pp. 911-918, 2007
- **Planqué**, **R.**, A. Dornhaus, N. Franks, T. Kovacs, J. Marshall, *Weighting waiting in collective decision making*, Behav. Ecol. Sociobiol. **61** (3), pp. 347-356, 2006

The project **Nonlinear estuarine hydrodynamics and environmental processes** extends a successful collaboration between B. Sommeijer en J. Huisman (Biology, IBED, UvA), in which the dynamics of phytoplankton populations in deep oceans was investigated. The present project focuses on spatial-temporal growth of phytoplankton, in which horizontal advection and diffusion processes are explicitly accounted for. The theory is also applied to estuaries, in which spatial variations in turbidity strongly affect the growth of phytoplankton.

- Huisman, J., **N.N. Pham Thi**, D.M. Karl and **B.P. Sommeijer**, *Reduced mixing generates oscillations and chaos in the oceanic deep chlorophyll maximum,* Nature 439, pp. 322-325, 2006
- **Zagaris A, A. Doelman, N.N. Pham Thi** and **B.P. Sommeijer**, *Blooming in a nonlocal, coupled phytoplankton-nutrient model*, SIAM Journal on Applied Mathematics 69 (4), pp. 1174-1204, 2008
- **Swart, H.E. de,** H.M. Schuttelaars and S.A. Talke, *Phytoplankton growth in turbid estuaries, a simple model*, Continental Shelf Research 29, pp. 136-147. Doi:10.1016/j.csr.2007.09.006, 2009
- Talke, S.A., **H.E. de Swart** and V.N. de Jonge, *An idealized model and systematic process study of oxygen depletion in highly turbid estuaries*, Estuaries and Coasts 32, pp. 602–620. Doi: 10.1007/s12237-009-9171-y, 2009

In 2009 S.C. Hille (UL) extended the collaboration with the biophysics group of Prof. T. Schmidt (UL) on **gradient sensing in Dictyostelium discoideum** by involving J. Dubbeldam (Mathematical Physics, TUD) on stochastic simulation of the signaling network. He intensified the collaboration with the Plant BioDynamics Laboratory (PBDL) at Leiden University on **reverse engineering of the auxin transport system in Arabidopsis**. Papers reporting on the established research results are in preparation and according to plan will be submitted in the fall of 2010. Moreover, a grant was obtained for the 'BetNet' interdisciplinary research proposal on **bet-hedging strategies in Bacillus subtilus** (with P. Haccou (CML, Leiden University), M. Emmerich (LIACS, Leiden University) and O. Kuipers (microbiology, RuG)) within the NWO Computational Life Sciences programme. The project started mid- December 2009.

- Boot, C.J.M., **S.C. Hille,** K.R. Libbenga and L.A. Peletier, *Polar auxin transport through Arabidopsis stems* (working title), in preparation
- Dubbeldam, J., **S.C. Hille** and T. Schmidt, *Gradient sensing in Dictyostelium discoideum in a noisy environment* (working title), in preparation

In genomics (notably SNP data), proteomics, and metabolomics, where the number of variables vastly exceeds the number of observations, the data call for **new statistical data-analysis methods for clustering, regression, and model fitting**. In an interdisciplinary collaboration, statisticians (Leiden University and Stanford University), collaborate with analytical chemists (LACDR, Leiden University and TNO, Zeist), biological psychologists (VU Amsterdam), and medical doctors and biostatisticians (Erasmus University, Rotterdam) to develop new methods and analyze new data at the same time.

- Damian D., M. Oresic, E. Verheij, **J.J. Meulman**, J. Friedman, A. Adourian, N. Morel, A. Smilde, J. van der Greef, *Applications of a new subspace clustering algorithm (COSA) in medical systems biology*, Metabolomics, 3, pp. 69-77, 2007
- Draisma, H.M., Th.H. Reijmers, F. van der Kloet, I. Bobeldijk-Pastorova, E. Spies-Faber, J.T.W.E. Vogels, **J.J. Meulman**, D.I. Boomsma, J. van der Greef, T. Hankemeier, *Equating, or Correction for Between-Block Effects with Application to Body Fluid LC−MS and NMR Metabolomics Data Sets*, Analytical Chemistry, 82, pp. 1039–1046, 2010

- Draisma, H.M., Th.H. Reijmers, I. Bobeldijk-Pastorova, **J.J. Meulman,** et al., *Similarities and differences in lipidomics profiles among healthymonozygotic twin pairs*, Omics, A journal of integrative biology, 12, pp. 17-31, 2008
- Rippe, R.C.A., P.H.C. Eilers, P.J. French, **J.J. Meulman**, *Statistical models for SNP genotyping en signal calibration*, Statistical Modelling, 10 (in press) , 2010

7.2. Cooperation with Dutch R&D institutions and industry

Among the many examples of collaboration with Dutch R&D institutions or industry we mention two, one from medicine and one from plasma technology.

The high risk of death for patients in the intensive care unit (ICU) necessitates careful metabolic control. ICU patients must be fed as soon as possible, with concomitant insulin and potassium administration to carefully regulate glucose and potassium levels. By combining knowledge from two fields: medicine and mathematics, Verbitskiy and the UMC Groningen expect to **develop new algorithms for comprehensive metabolic regulation in ICU**, which will further improve the quality of care and safety of critically ill patients.

- Hoekstra, M., M. Vogelzang, **E. Verbitskiy**, M.W.N. Nijsten, *Health technology assessment review: Computerized glucose regulation in the intensive care unit – How to create artificial con- trol?*, Critical Care 13:223, 2009
- Hoekstra, M., M. Vogelzang, **E. Verbitskiy**, M.W.N. Nijsten, *Hourly measurements not required for safe and effective glycemic control in the critically ill patient*, Critical Care 14:404, 2010

Streamers are generic nonlinear phenomena in electric discharges, in lightning as well as in a multitude of technical applications. This is because they efficiently convert pulsed electrical energy into chemical products in air. (You smell the ozone when you attend an experimental demonstration on dischanges and lightning in a science museum.) This mechanism can be used for air purification, sterilization, biofuel cracking, etc. It is one of the focal points of the new STW-programme ‘Building on Transient Plasma’, in which mathematicians and physicists collaborate with 16 industrial partners and research institutes on these questions. The streamers achieve this effect by focusing the electric fields in small regions at their tips; there electrons are accelerated to high energies very far from equilibrium, and excite ambient molecules in an efficient manner. The challenge of nonlinear theory lies in understanding field focusing and electron acceleration, in technical discharges as well as in natural lightning.

- *The multiscale nature of streamers*, **U. Ebert**, **C. Montijn**, **T.M.P. Briels, W. Hundsdorfer**, **B. Meulenbroek**, **A. Rocco**, E.M. van Veldhuizen, Plasma Sources Science and Technology **15**, S118-S129, 2006
- *Streamers, sprites, leaders, lightning: from micro- to macroscales, an editorial review and introducton to the cluster issue on "Streamers, Sprites and Lightning"* in J. Phys. D, **U. Ebert**, D.D. Sentman, J. Phys. D: Appl. Phys. **41**, 230301, 2008
- *Positive and negative streamers in ambient air: measuring diameter, velocity and dissipated energy*, T.M.P. Briels, J. Kos, G.J.J. Winands, E.M. van Veldhuizen, **U. Ebert**, J. Phys. D: Appl. Phys. **41**, 234004, 2008
- *Positive and negative streamers in ambient air: modeling evolution and velocities*, A. Luque, V. Ratushnaya, **U. Ebert**, J. Phys. D: Appl. Phys. **41**, 234005, 2008

7.3. Transfer researchers and students to Universities

The following PhD students and postdocs were active at some during the reporting period, and have found their way to different academic environments.

- Asadi, Esmaeel (VUA) PhD: mathematics, IPM, Iran
- Ashyraliyev, Maksat (CWI) PhD: mathematics, Bahcesehir University, Turkey
- Balint, Andras (VUA) PhD: mathematics, Gothenburg, Sweden
- Bierkens, Joris (UL) PhD: mathematics, CWI and UL
- Blanchère, Sébastien (TU/e) postdoc: mathematics, Université d'Aix-Marseille 1, France
- Brau, Fabian (CWI) postdoc: physics, Mons-Hainaut, Belgium
- Briels, Tanja (TU/e) PhD: physics, Fontys Hogeschool, Eindhoven
- Bruggeman, Frank (CWI) postdoc: life sciences, UvA
- Castillo, Ismael (VUA) postdoc: mathematics, Paris VI, France
- Cheliotis, Dimitris (TU/e) postdoc: mathematics, Athens, Greece

- Donauer, Stefanie (VUA) PhD: mathematics, Universität Hannover, Germany
- Dubinkina, Svetlana (CWI) PhD: physics, Louvain-la-Neuve, Belgium
- Dusseldorp, Elise (UL) postdoc: statistics, TNO
- Eberard, Damien (RuG) postdoc: mathematics, Université de Lyon, France
- Efstathiou, Konstantinos (RuG) postdoc: mathematics, RuG
- Elrofai, Hala (VUA) PhD: mathematics, Khartoum, Sudan
- Es-Sarhir, Abdelhadi (UL) postdoc: mathematics, TU Berlin, Duitsland
- Fey, Anne (VUA) PhD: mathematics, CWI/VUA
- Fortney, Jon Pierre (RuG) postdoc: unknown
- Gennip, Yves van (TU/e) PhD: mathematics, Simon Fraser University, Canada
- Hazewinkel, Jeroen (CWI) PhD: oceanography, Scripps Institution for Oceanography, UC SanDiego, USA
- Heijster, Peter van (CWI) PhD: mathematics, Brown University, USA
- Hlod, Andriy (TU/e) PhD: mathematics, TU Eindhoven
- Hupkes, Hermen Jan (UL) PhD: mathematics, Brown University, USA
- Khanh, Nguyen Huu (UvA) postdoc: academia, Vietnam
- Kooij, Anita van der (UL) PhD: psychology, UL
- Kramar, Miro (VUA) PhD: mathematics, Rutgers, USA
- Kruijer, Willem (VUA) PhD: mathematics, Université Paris-Dauphine, France
- Li, Chao (CWI) PhD: physics, TU/e, NL
- Linting, Marielle (UL) PhD: sociology, UL
- Lombardo, Sara (VUA) postdoc: mathematics, University of Leeds, UK
- Lukina, Olga (RuG) PhD: mathematics, University of Leicester, UK
- Luque, Alejandro (CWI) postdoc: astrophysics, Granada, Spain
- Maillard, Gregory (TU/e) postdoc: mathematics, Université d'Aix-Marseille 1, France
- Meulenbroek, Bernard (CWI) PhD: mathematics, TUD
- Muskulus, Michael (UL) PhD: mathematics, NTNU, Trondheim, Norway
- Palla, Luigi (VUA) postdoc: unknown
- Pasquotto, Federica (VUA) postdoc: mathematics, VUA
- Pétrélis, Nicolas (TU/e) postdoc: mathematics, Nantes, France
- Pijl, Sander van der (CWI) postdoc: searching for a position
- Polyuga, Rostyslav (RuG) PhD: mathematics consultancy, TU/e
- Röger, Matthias (TU/e) postdoc: mathematics, TU Dortmund, Germany
- Roquain, Etienne (VUA) postdoc: mathematics, Paris VI, France
- Savcenco, Valeriu (UvA) PhD: mathematics, TU/e
- Setiawan, Adi (VUA) PhD: mathematics, Universitas Kristen Satya Wacana,
- Subramanian, Easwar (RuG) PhD: mathematics, INRIA Sophia Antipolis, France
- Sun, Rongfeng (TU/e) postdoc: mathematics, Singapore
- Terra, Guido (UvA) PhD: academia, Utrecht University College
- Trapman,Pieter (VUA) postdoc: mathematics, Stockholm, Sweden
- Veneroni, Marco (TU/e) postdoc: mathematics, TU Dortmund, Germany
- Wieringen, Wessel van (VUA) postdoc: mathematics/medicine, VUA
- Wojtylak, Michał (VUA) postdoc: mathematics, Jagiellonian University, Poland
- Zagaris, Antonios, (CWI/UvA) postdoc: mathematics, UT

7.4. Transfer researchers and students to industry

Some PhDs and postdocs have left academia for positions in private companies:

- Banachewicz, Konrad (VUA) PhD: an investment bank
- Brok, Wouter (CWI) postdoc: OTB Solar
- Driesse, Ramon (UvA) PhD: currently searching a position
- Gillett, Alexis (VUA) PhD: Statkraft
- Guyonne, Vincent (VUA) PhD: Dassault
- Holtman, Sijbo (RuG) PhD: Aegon, Den Haag

- Krottje, Johannes (CWI) PhD: mathematics, ABP Pensioenfunds
- Panhuis, Peter, Peter in 't (TU/e) PhD: Shell Rijswijk
- Pham Thi, Nga (CWI) PhD: mathematics, Rabobank
- Ploeg, Harmen van der (UvA) PhD: software engineer Dentline B.V.
- Svensson, Christian (UL) PhD: Sampension (a pension insurance company), Kopenhagen
- Valkhoff, Nienke (CWI) PhD: Manager Training & Education at Incontrol Enterprise Dynamics
- Vagvolgyi, Balint (VUA) PhD: NIBC Bank
- Venkatraman, Aneesh (RuG) PhD: Royal Bank of Scotland, UK
- Wójcik, Wojciech (VUA) PhD: Motorola
- Zareba, Pawel (VU) PhD: a bank
- Zmarrou, Hicham (UvA) PhD: Nederlands Lucht- en Ruimtevaartlaboratorium

7.5. Transfer researchers and students to other activities

A few former PhD students and postdocs are currently in other positions:

- Montijn, Carolynne (CWI) PhD: NLR, NL
- Raalte, Marc van (CWI) postdoc: Energie Centrum Nederland
- Rotten, Bart van de (UL) postdoc: high school teacher mathematics
- Sitters, Harry (RuG/FI) PhD: leraar in onderzoek
- Spitoni, Cristian (TU/e) postdoc: Leids Universitair Medisch Centrum
- Vondenhoff, Erwin (TU/e) PhD: Centraal Bureau voor de Statistiek

8. Added value for other disciplines, Dutch R & D institutions, industry, and society at large

A number of large-scale multidisciplinary workshops have been, and still are, organized by NDNS$^+$ that attract a lot of attention from other sciences and from industry, enhance ongoing cooperation and also contribute to follow-up activities. Here the workshops 'Mathematics of Life Sciences' (RUG, 2005) organized by the NDNS$^+$ board Broer, Doelman, Van der Vaart and Verduyn Lunel) should be mentioned, as well as 'Mathematics of Earth Sciences' (RUG, 2006) organized by Broer and Doelman and the IMAU members Henk Dijkstra and Huib de Swart. Also we should mention the workshop 'Statistics in the Life Sciences' (RUG, 2009) organized by the NDNS$^+$ members Wit and Van den Heuvel and by the biostatistician Siem Heisterkamp; the latter two are both adjunct professors RUG-Schering Plough, where notably the position of Van den Heuvel was made possible by NDNS$^+$. Also many activities under the title 'Dynamics of Patterns', e.g. seminars and workshops organized by Rademacher, Snoeijer, Ebert and Hek should be mentioned, since they also triggered and stimulated interdisciplinary research.

The cooperation stimulates many successful interdisciplinary joint proposals in the various NWO themes, e.g., ELS, CLS, CMSB,... as wel as in EU projects (e.g. Seventh-Framework ITN "FIRST" (Project 238702), or "CON4COORD" (Project 223844)). Apart from mathematicians, also physicists and computer scientists cooperate in the research. The research partners are from earth and life science (including medical) institutions both within academia and from outside (e.g., the IMAU, KNMI, NIOZ, the UMCG,...). A number of these projects have already led to successful PhD defences with publications both within mathematics and outside.

Further examples of interaction between mathematics and other disciplines include the mathematics-physics collaboration in Groningen (Broer, Van Enter, Efstathiou, Kueslke, Waalkens and Takens (emeritus-RUG)) and Leiden (Den Hollander, Redig - later RUN), which is supported by several guests (Cushman (Calgary), Fernandez (Rouen, later UU), and Sadovskii (Dunkerque)). Here also the adjunct professor Verbitskiy (RUG-Philips, later RUG-UL) plays a role. At VUA new contacts have been made with NXP leading to two joint publications on MEMS resonators and switches, and Ebert's group at CWI is involved in 6 projects (5 STW and 1 SenterNovem) with 15 partners in industry and research institutes.

In addition the cluster has co-initiated two multidisciplinary programs at NWO itself: together with Wim van Saarloos (physics, Leiden), Arjen Doelman has initiated and coordinated the NWO-FOM program ‘Dynamics of Patterns’. In the context of this program, four PhD and four postoc projects have been funded at the intersection of (nonlinear) mathematics and physics (the budget was € 1.400.000). The ‘Complexity’ program, that is supported by the majority of all NWO divisions, can be seen as a successor of this program. Mathematics, and especially the $NDNS^+$ cluster, has played a central role in starting up this program (through Doelman). Embedded in this program, there is an explicit cooperation with industry – the Dutch National Bank (Nederlandse Bank), the Dutch Railway (Nederlandse Spoorwegen) and several ICT companies. Approximately 30 PhD/postdoc projects will be funded by the Complexity program (the budget is about € 7.000.000).

9. Outreach activities

A series of Study Groups Mathematics with Industry has been organized by different insitutes (2005: VU, 2006: Eindhoven, 2007: Utrecht, 2008: Twente, 2009: Wageningen, 2010: CWI), in which cluster members have played active roles both as organisers and as participants. This highly successful activity brings together academic mathematicians with commercial companies and other non-academic organizations in a non-standard way. During a single, highly-paced week, the participants apply mathematical tools to solve practical problems that have been presented by the participating industries.

The Study Groups are a success on many different levels. At the basic level, a surprisingly large number of the problems are solved within that single week, to the satisfaction of both industrialists and academics. At the same time this yearly week provides essential training to Masters and PhD students in the Netherlands and outside, in which they learn techniques of modelling, analysis, and communication in a hands-on manner. At an even higher level, the long-term effect of bringing together academics with industrialists leads to long-term collaborative research. Examples of this are the PhD project of Hlod (inspired by the Study Group at TUD in 2004) and various publications in scientific journals of the results of the week, e.g. [70][111]. The success of the mathematical Study Group has inspired the physics community in the Netherlands to organize its first ‘Study Group Physics and Industry’ in 2010.

Note that although $NDNS^+$ and its members tend to play central roles in the organization of these weeks, a fact which is related to the subject area of $NDNS^+$, the Study Group organizations always aim to cover as much of mathematics as possible: there is no content bias.

An intensive outreach is ongoing in the direction of secondary schools and their teachers, which is of importance for restoring the quality of the connection with the universities. Here Henk Broer, Joost Hulshof, Mark Peletier have been and are still involved (OCW-ministerial committees cTWO, NLT, Nationale Wiskundedagen). Many other cluster members contributed to master classes and seminars for secondary schools (including the ‘Vacation course’ for teachers).

The CWI work on lightning has a broad appeal to the public. During the course of the cluster, her group’s work has appeared twice on TV, 5 times on the radio, 5 times in newspapers (NRC, Volkskrant, Telegraaf), 6 times in Dutch publications addressing high school students and other scientifically interested laypersons, and 10 times in journals of scientific societies.

Part B: Cluster research

The research of the cluster NDNS$^+$ is organized in six themes: Bifurcations and Chaos, Networks and Delays, Scientific Computing, Transient Dynamics and Multiple Scales, Patterns and Waves, and Statistics. These subdivisions are to be viewed as overlapping groups of activities, where most people and most activities belong to several themes simultaneously. Nonetheless, each of the themes has clearly defined research lines, and in the sections below we describe the progress and developments of each of them separately.

Following the research descriptions we detail the national and international collaborations and the additional activities for the cluster as a whole, to prevent unnatural allocation to separate themes of what are essentially multi-theme and often cluster-wide features.

The cluster NDNS$^+$ has been constructed around the interaction between mathematics on one hand and the applications in the earth and life sciences on the other. Many of the cluster activities are inspired by questions arising from applications, and there are many close collaborations between cluster members and non-mathematicians (see also Section 7 of Part A). We therefore end this part of the evaluation by describing the develoment of the interrelationships between mathematics and applications.

Theme: Bifurcations and Chaos

Introduction

The subject of bifurcations and chaos is at the heart of (finite-dimensional) dynamical systems theory as this was formed over the second half of the last century and has inspired so many researchers working in other fields. One *raison d'être* of the theory is that it provides robust dynamical modelling in a huge number of application areas, varying from mechanics, physics, chemistry, and economics to the earth and life sciences. In particular it provides a good way to model the transitions from rather simple, say periodic, to more complex dynamics, say, to multi- or quasi-periodic and chaotic dynamics. Although the theory is at its most powerful for finite-dimensional systems, its ideas have widely spread over areas like Partial Differential Equations and other infinite-dimensional situations, where the background language has proved extremely successful.

1. People

- CWI: Dr. J. Rademacher
- RuG: Prof. dr. H.W. Broer, prof. dr. F. Takens, prof. dr. G. Vegter, prof. dr. H. Waalkens
- UL: Dr. O.W. van Gaans, dr. S.C. Hille, prof. dr. S M. Verduyn Lunel
- UT: Prof. dr. S.A. van Gils
- UU: Prof. dr. O. Diekmann, dr. H. Hanßmann, dr. Y. Kuznetsov
- VUA: Dr. B. Rink

During the reporting period H. Waalkens was promoted to full professor.

2.1. Description of the research theme

Quasi-periodic bifurcation theory somewhat resembles the bifurcation theory of equilibria and periodic orbits in that the bifurcation scenario in the product of phase space and parameter space has a roughly similar geometry, but a dense set of resonances gives rise to directions that become like Cantor sets. The most interesting chaotic dynamics takes place in the gaps of these.
The global KAM Theory in this same spirit led to a 'Cantorisation' of the Lagrangean torus bundle, for which still all kinds of geometrical invariants like monodromy can be defined. The mildly degenerate Hopf-Neimark-Sacker bifurcation involves the complication of Arnold resonance tongues, thereby creating flames and pockets [26]. It combines singularity theory with numeric and symbolic

work as well as computer graphics. The latter also holds for the life science and climate modelling, combined with a thorough understanding of the background mathematics. In the life-science models a predator-prey model was analyzed, as well as neuronal coupled cell systems with delays. In the former case we found multiple periodic orbits and in the latter unstable attractors connected by heteroclinic cycles [25]. In the climate models we look for qualitative models to explain climate variability of the Northern Atlantic Ocean. Here we find quasi-periodic period doublings and quasi-periodic Hopf bifurcations together with strange attractors [106].

2.2. Development of the research theme

The developments of the subthemes during the cluster period in all cases amount to real innovation. This holds for the KAM Theory where in the bifurcations we extended to the case of multiple normal eigenvalues which enables to deal with an example like the quasi-periodic Hamiltonian-Hopf bifurcation. The set-up is now a mixture of singularity theory with KAM Theory. The global KAM Theory was redeveloped from scratch using methods from differential geometry. It should be noted that his theme has an important outreach to theoretical physics and chemistry, where the corresponding quantum monodromy explains certain spectral defects. The mildly degenerate Hopf-Neimark-Sacker bifurcation forms a 'next' case in the theory of bifurcations.

There also has been significant progress on a variety of global bifurcations, which are essential for studies of pattern formation and thus for a range of applications in natural systems. This aspect was not forseen in the original description of the theme's research. Examples are a study of Twist maps on the plane that lead to a novel chaos-forcing criterium [11], and key results in systems with symmetry or reversibility, which explain their role in chaotic dynamics [62].

2.3. Most important research highlight of the theme

The RUG-IMAU climate research focuses on parametrized PDE models of the Northern basin of the Atlantic Ocean, including the atmosphere. By a Galerkin projection ODE models were derived in spaces that have around 50 dimensions, and sometimes more than 70. The exploration needs significant computer skills for which we collaborate with external parties. By further reducing to a center manifold we found plausible explanations for the Atlantic Multidecadal Oscillation, where the mathematical cause turns out to be a Hopf Saddle-Node Bifurcation. We detected many Hénon-like strange attractors and a large set of quasi-periodic bifurcations. Motivated by this as well as by earlier meteorological results, fundamental research was triggered of standard models of the Hopf Saddle-Node in three dimensional space, also for maps. Thus an even better understanding was obtained of the phenomena in the Galerkin projections.

- **Sterk, A.E., R. Vitolo, H.W. Broer**, C. Simò and **H.A. Dijkstra**, *New nonlinear mechanisms of midlatitude atmospheric low-frequency variability*, Physica D: Nonlinear Phenomena 239, pp. 701-718; DOI information: 10.1016/j.physd.2010.02.003, 2010

2.4. Five key publications

- **Berg, J.B. van den, R.C. van der Vorst,** and **W. Wojcik**, *Chaos in orientation reversing twist maps of the plane,* Topology and its Applications 154, pp. 2580-2606, 2007
- **Broer, H.W** and **H. Hanßmann** and J. Hoo, *The quasi-periodic Hamiltonian Hopf bifurcation*, Nonlinearity 20, pp. 417-460, 2007
- **Broer, H.W.,** C. Simò and **R. Vitolo**, *The Hopf-Saddle-Node bifurcation for fixed points of 3D-diffeomorphisms, analysis of a resonance `bubble'*, Physica D, 237, pp. 1773-1799, 2008
- **Broer, H.W., S.J. Holtman, G. Vegter, and R. Vitolo**, *Geometry and dynamics of mildly degenerate Hopf-Neimarck-Sacker families near resonance*, Nonlinearity 22, pp. 2161-2200, 2009
- **Homburg, A.J.** and J. Knobloch, *Multiple homoclinic orbits in conservative and reversible systems,* Transactions Amer. Math. Soc. 358, pp. 1715-1740, 2006

Theme: Networks and delays

Introduction

Cell activity involves a huge regulatory network of protein interactions driven by external physical and chemical signals in which the physical processes themselves influence the way in which the information coded in the genes is used. This leads to complicated dynamical systems with feedback loops. It is very natural that there are time delays in the feedback loop and this leads to an infinite dimensional dynamical system. Although at a different scale, similar infinite dimensional dynamical systems arise to describe physiologically structured populations. These last models are developed to tackle challenging questions, like whether one can predict catastrophic shifts in ecosystems from observable spatial patterns. A difficult problem in modeling the behavior of networks is that they often consist of huge numbers of 'nodes' but cannot be described by continuous models. This leads to models based on cellular automata, coupled map lattices and lattice differential equations. The analysis of such models leads to fundamental questions in the theory of functional analysis and dynamical systems.

1. People

- CWI: Dr. J. Rademacher
- UU: Prof. dr. O. Diekmann
- UT: Prof. dr. S.A. van Gils
- UL: Prof. dr. S.M. Verduyn Lunel, dr. O.W. van Gaans, dr. S.C. Hille
- VUA: Dr. B. Rink

2.1. Description of the research theme

In the field of PDEs, travelling waves have played a major role in the development of the theory. The most important cause for this is the simple fact that existence and stability questions for wave solutions reduces the PDE which is an infinite dimensional dynamical system to a finite dimensional ordinary differential equation. Since the theory for ODEs is rich and well-developed, this reduction is very powerful.
It is natural to follow the same approach when considering models based on cellular automata, coupled map lattices, and lattice differential equations. However, the travelling-wave reduction now no longer leads to an ODE, but to a functional differential equation of mixed type (MFDE). Here the word 'mixed' refers to the fact that the derivative in this differential equation depends on both retarded and advanced arguments. The understanding of such dynamical systems has long been severly limited due to the fact that it is ill-posed as an initial value problem and hence admits no solution (semi)flow.
A major challenge of this theme is to provide a rigorous mathematical background that can confirm and predict some of the phenomena that we have already observed numerically. Faced with a specific dynamical system, one would ideally want to find and classify the different generic types of behaviour that can arise and divide the set of initial conditions accordingly. In general, we are still far removed from this goal at present, but nevertheless a number of deep results have been already obtained that uncover some aspects of the specific behaviour of lattice differential equations, such as propagation failure of travelling waves through discrete media.

2.2. Development of the research theme

A first theme is the study of lattice differential equations. These are continuous-time infinite dimensional dynamical systems, which possess a discrete spatial structure modeled on a lattice. Such equations play an important role in modeling a variety of applications with spatial structure and can be found in chemical reaction theory, image processing and pattern recognition, material science and biology. On the fundamental side, via novel techniques some classical problems in the study of Fermi-Pasta-Ulam lattice dynamics have finally been resolved [96].

A second theme is the study of the existence of invariant measures for stochastic functional differential equations. In order to apply methods from ergodic theory to study the limit behavior of the distribution of solutions of such equations, it is necessary to have the existence of an invariant measure (or stationary distribution). Existing results regarding the existence of an invariant measure for stochastic dynamical systems either require strong smoothness assumptions or a dissipativity condition on the underlying deterministic flow. However, for stochastic functional differential equations, these conditions fail dramatically, and proving the existence of an invariant measure is a challenging problem. Our main result avoids smoothness or dissipativity and is based on exponential dichotomies [19]. This project is part of a VIDI project by Van Gaans.
A third theme concerns the modelling, analysis and simulation of dynamical systems that are related to self-organisation and chemotaxis with emphasis on a functional analytic understanding of approximation of dynamical systems, e.g. discrete systems by continuum models, limits related to singular perturbation such as diffusion and hydrodynamic limits [98].

The ambition of the orginal research plan was to bridge the research communities devoted to, respectievely, finite- and infinite-dimensional dynamical systems, in order to jointly successfully analyse the huge regulatory network of protein interactions that become presently available. Indeed, the analysis of models based on cellular automata, coupled map lattices, and lattice differential equations have turned out to be instrumental in bringing together the expertise from finite- and infinite-dimensional dynamical systems, but the basic analysis of such models lead to deep questions in the theory of functional analysis and dynamical systems which took more time than anticipated. This resulted in that we did not come as far as we hoped to in applying our joint expertise in the analysis of the regulatory networks of protein interactions. The next step is left for the future.

2.3. Most important research highlight of the theme

It was only during the last decade that linear functional differential equations of mixed type (MFDE) were shown to admit Fredholm properties and exponential dichotomies [76]. Building upon these results, Hupkes and Verduyn Lunel, were able to consistently write down the fixed-point problem that lies at the heart of the construction of invariant manifolds. Our techniques were inspired by Mielke, who was faced with the absence of a semiflow in his work on elliptic PDEs. Following the approach developed by Vanderbauwhede et al., this allowed us to develop a key tool for general nonlinear MFDEs. In particular, we proved that the dynamics close to equilibria and periodic solutions can be fully captured by an ODE on a finite-dimensional center manifold. This has finally opened the rich classical dynamical systems toolbox for use in our infinite-dimensional setting. Based upon such a center-manifold reduction, we can already find existence results, apply Lin's method, and perform bifurcation analysis for infinite-dimensional dynamical systems in economic theory, elasticity and solid state physics.

- Hupkes, H. J., S.M. Verduyn Lunel, *Center manifold theory for functional differential equations of mixed type*, J. Dynam. Differential Equations 19, no. 2, pp. 497-560, 2007

2.4. Five key publications

- **Hille, S.C.**, *Local well-posedness of kinetic chemotaxis models*. J. Evol. Equ. 8, no. 3, pp. 423-448, 2008
- **Hupkes, H.J.**, E. Augeraud-Véron, and **S.M. Verduyn Lunel**, *Center projections for smooth difference equations of mixed type*. J. Differential Equations 244, no. 4, pp. 803-835, 2008
- **Hupkes, H.J.** and **S.M. Verduyn Lunel**, *Lin's method and homoclinic bifurcations for functional differential equations of mixed type*, Indiana Univ. Math. J. 58, no. 2, pp. 2433-2488, 2009
- **Rink, B.**, *Proof of Nishida's conjecture on anharmonic lattices*, Comm. Math. Phys. **261**, pp. 613-627, 2006
- **Bierkens, J., O. van Gaans,** and **S. Verduyn Lunel**, *Existence of an invariant measure for stochastic evolutions driven by an eventually compact semigroup*. J. Evol. Equ. 9, no. 4, pp. 771-786, 2009

Theme: Scientific Computing

Introduction

Scientific computing is the mathematical discipline par excellence concerned with approximation of and the extraction of quantitative information from complex systems. For many nonlinear partial differential equations of practical importance, numerical approximation with computers is the prime method of obtaining information about the solutions. Numerical simulations are used to explore and gain intuition on the behavior of dynamical systems and to validate analytical results based on asymptotic assumptions. Numerical algorithms are a workhorse of science and engineering, where they are used for prediction and forecasting, as a cheap alternative to laboratory experiments, and as a software tool to obtain precise answers to applied questions. Climate simulations and astrophysical simulations allow us to study the dynamics of the earth or the universe on time scales that are unattainable by direct observation. Seismic inversion allows us to look deep into the earth's crust at a relatively small cost.

1. People

- CWI: Dr. J.G. Blom, prof. dr. J. Frank, prof. dr. W. Hundsdorfer, prof. dr. C. Oosterlee, dr. B.P. Sommeijer, prof. dr. J.G. Verwer,
- RUG: Prof. dr. A.E.P. Veldman, dr. ir. R.W.C.P. Verstappen, dr. ir. F.W. Wubs
- TUD: Prof. dr. A.W. Heemink, prof. dr. J.M.A.M. van Neerven, prof. dr. ir. C. Vuik
- TU/e: Prof. dr. E.H. van Brummelen (TU/e).
- UvA: Dr. J.A. Brandts, prof. dr. R.P. Stevenson
- UT: Prof. dr. O. Bokhove, dr. M.A. Botchev, prof. dr. B.J. Geurts, prof. dr. J.J.W. van der Vegt
- UU: Dr. P. Zegeling

A number of participants within the scientific computing theme were promoted to full professor during the evaluation period: Stevenson (UvA), Oosterlee (TUD), Hundsdorfer (RUN), van Brummelen (TU/e), and Frank (UvA).

2.1. Description of the research theme

Scientific computing research for dynamical systems falls naturally into two categories. The first of these is approximation theory, defined in the limit as the discretization mesh tends to zero for a fixed interval. Considerations such as accuracy, convergence, error estimation and adaptivity are characteristic. The second category is that of discrete dynamical systems theory, and pertains to the case of fixed discretization mesh and recursive application of the numerical map. Considerations in this category are long-time stability, invariant and attracting sets, and qualitative solution aspects such as discrete conservation laws, symmetries, and monotonicity principles.

To solve partial differential equations accurately and efficiently, numerical methods must be adaptive. This requires a local (a posteriori) error estimator, a mesh refinement strategy, and efficient iterative solvers. A significant amount of work in the cluster has addressed these issues.

Discrete dynamics is important for problems for which pointwise accurate solution is impossible due to chaotic error growth. Long-time simulations of molecular systems, turbulent fluids, or the earth's climate are performed, not to make precise predictions about the solution of a given initial value problem, but rather to generate a data set for statistical analysis.

2.2. Development of the research theme

Developments in approximation theory include work of Brandts (UvA) on simplicial partitions, with importance to mesh quality in finite element methods, which was published in SIAM Review [21]; of Van der Vegt (UT) on the development and analysis of discontinuous Galerkin finite element methods with applications in compressible, incompressible and two-phase flows; of Zegeling (UU) on adaptivity using moving mesh methods, culminating in the thesis [32], and of van Brummelen on goal-oriented error estimation and optimal adaptivity for free-boundary and coupled problems [116] .

Developments in the dynamical aspects of numerical methods include work by Verstappen (RUG) on energy-enstrophy discretizations of fluids [113]. Frank (CWI) showed that statistical mean field obtained from long time simulations with such methods is highly dependent on the discrete conservation laws and time symmetry [43].

Within the original proposal, a major subtheme has been the further development of path following techniques that can (automatically) trace manifolds of bifurcations. In this context, novel ideas and methods have been developed by Broer, Kutznetsov, and Rademacher – who introduced and extended these techniques to the computation of absolute and essential spectra associated to spatially periodic patterns.

2.3. Most important research highlight of the theme

A long sought-after fundamental convergence result shows that an adaptive finite-element method based on a-posteriori error estimates yields convergence with optimal complexity for elliptic partial differential equations. This is a very powerful result, and it is the first on convergence and optimality of adaptive finite-element methods. The methodology relies on a carefully selected combination of the error-estimation procedure and the refinement strategy. It draws on several important themes in numerical analysis.

- **Stevenson, R.P.**, *Optimality of a standard adaptive finite element method*, Found. Comput. Math. 7, no. 2, pp. 245-269, 2007

2.4. Five key publications

- **Rademacher, J.D.M.**, B. Sandstede, A. Scheel, *Computing absolute and essential spectra using continuation*, Physica D 229, pp. 166-183, 2007
- Lang, J. and **J.G. Verwer**, *On global error estimation and control for initial value problems*, SIAM J. Sci. Comput. 29, pp. 1460-1475, 2007
- Erlangga, Y.A., **C.W. Oosterlee** and **C. Vuik**, *A novel multigrid based preconditioner for heterogeneous Helmholtz Problems*, SIAM J. Sci. Comput.27: pp. 1471-1492, 2006
- Niet, A.C. de, F.W. Wubs, **H.A. Dijkstra**, and A. Terwisscha van Scheltinga, *A tailored solver for bifurcation analysis of ocean-climate models*, J. Comput. Phys. 227(1), pp. 654-679, 2007
- **Zee, K.G. van der, E.H. van Brummelen** and R. de Borst, *Goal-oriented error estimation and adaptivity for free-boundary problems: The shape-linearization approach*, SIAM J. Sci. Comput. 32, pp. 1093–1118, 2010

Theme: Transient Dynamics and Multiple Scales

Introduction

Many of the models arising in the earth and life sciences (and also in other areas of science and engineering) have a multiscale structure. It is for this reason that this research theme has been central to the cluster. Although a bit less obvious, the same can be said of 'transient dynamics'. The well-developed theory of asymptotic stability and/or attractivity lies at the core of dynamical systems theory. In nature, however, one cannot take the limit $t \to \infty$: structures will only be 'stable' on a certain timescale, beyond which other aspects of the model take over. Hence, in realistic models stability and attractivity can only be considered to be transient phenomena.

1. People

- CWI: Prof. dr. U. Ebert, prof. dr. J. Frank, dr. J.G. Blom, dr. D. Crommelin, dr. J. Rademacher, dr. R. Merks
- TU/e: Prof. dr. M.A. Peletier, dr. A. Muntean, dr. I.S. Pop
- UL: Prof. dr. A. Doelman, dr. V. Rottschäfer
- UT: Prof. dr. S.A. van Gils, dr. O. Bokhove, dr. A. Zagaris
- VUA: Prof. dr. G.J.B. van den Berg, prof. dr. J. Hulshof

A number of participants within the theme Transient Dynamics and Multiple Scales were promoted to professor during the reporting period: Van Gils (UT), Frank (UvA), van den Berg (VU).

2.1. Description of the research theme

The research in the cluster has focused on low-dimensional 'coherent structures', as exhibited by realistic models in meteorology, oceanography, neurophysiology, plasma technology etc. Coherent structures are often not stable in the classical asymptotic sense (i.e., for all time), they may appear, exist for some (long) time, and eventually disappear. This continuous transient process of emergence, slow evolution, and annihilation is remarkably subtle, but several aspects of it can be studied mathematically in reduced settings. It is naturally related to the multiple-scale nature of the model. This sub-area was, and still is, especially promising, since there is an intimate relationship between experimental observations, computer simulations, modeling, numerical analysis and dynamical systems theory. Another feature of multi-scale problems that has been the subject of study within the cluster are the hierarchies of length and time scales that should be eliminated subsequently, leading to a hierarchy of reduced models. Examples of such a hierarchy are moving fronts and interface approximations.

2.2. Development of the research theme

As envisioned at the onset of the cluster, the research within this theme has developed strongly, and for a large part along the lines sketched above. For instance, together with co-workers, Doelman took a number of significant steps in the direction of understanding the evolution of interacting localized structures [40][37][55]. The Eindhoven group gave the first rigorous characterization of curved stripe patterns in block copolymers [87] and made various advances in the homogenization of systems with non-trivial cell-problem behaviour [78][79], and the group at the VUA made significant progress in the understanding of the different scales in singularity formation in several (geometric) PDEs and free boundary problems [16][17].

A development that was not anticipated is the success of the work by Ebert on the multiscale nature of streamers, sprites, and lightning (see below). This is truly an exciting new research area in which observations, modelling, analysis, numerical analysis, and simulations go hand in hand. Moreover, this area is highly relevant from the point of view of technological applications, as well as within the earth sciences.

Another unanticipated development is the research performed by young scientists that have joined the cluster through a position pre-financed by the cluster. The work of Crommelin on metastable atmospheric regimes [75] and that of Rademacher on the dynamics of coherent structures in evolutionary equations [91] are prime examples of this state of affairs. Yet another example is the work of Zagaris on algorithmic reduction of high-dimensional dynamical systems to low-dimensional flows [115].

2.3. Most important research highlight of the theme

A thunderstorm generates electric fields inside, between, below, and above thunderclouds, and these fields decay algebraically in space. The air density in the atmosphere decreases exponentially with altitude, and so does the break-down electric field. It therefore was predicted already in 1925 that enormous discharges could appear above thunderclouds, but they were observed only in 1990. They are now known as sprites, halos, elves etc. Sprites consist of complex trees of streamer fingers, and each streamer is surrounded by a thin layer of electrical charge that suppresses the electric field in the interior and strongly enhances it at the tip; this is where the finger grows. By combining analytical solutions and simulations with adaptive grid refinement, we could study the evolution of halos and sprites above a thundercloud, taking all relevant features into account, from inner scales of meters up to the 90 km distance from ground to ionosphere. The results fit geophysical observations very well and constitute a major step in quantitatively understanding atmospheric discharges.

- **Luque, A., U. Ebert**, *Emergence of sprite streamers from screening-ionization waves in the lower ionosphere,* Nature Geoscience 2, pp. 757-760, 2009

2.4. Five key publications

- Budd, C.J., **V. Rottschäfer**, J.F. Williams, *Multibump, blow-up, self-similar solutions of the complex Ginzburg-Landau equation*, SIAM J. Appl. Dyn. Syst. 4, no. 3, pp. 649-678, 2005
- Cagnan, H., H.G.E. Meijer, **S.A. van Gils**, M. Krupa, T. Heida, M. Rudolph, W.J. Wadman, H.C.F. Martens, *Frequency-selectivity of a thalamocortical relay neuron during Parkinson's disease and deep brain stimulation: a computational study,* European Journal of Neuroscience, 30(7) pp. 1306-1317, 2009
- Dubinkina, S., **J. Frank**, *Statistical mechanics of Arakawa's discretizations*, J. Comput. Phys. 227, no. 2, pp. 1286-1305, 2007
- Majda, A.J., C. Franzke, **D. Crommelin**, *Normal forms for reduced stochastic climate models*, Proc. Natl. Acad. Sci. USA 106, no. 10, pp. 3649-3653, 2009
- **Zagaris, A., A. Doelman, N.N. Pham Thi, B.P. Sommeijer**, *Blooming in a nonlocal, coupled phytoplankton-nutrient model*, SIAM J. Appl. Math. 69, no. 4, pp. 1174-1204, 2009

Theme: Patterns and Waves

Introduction

Pattern formation is a very apparent phenomenon in the natural sciences, often immediately recognizable. To explain and predict these phenomena, however, is a major scientific challenge. In this theme we address this challenge.

1. People

- CWI: Prof. dr. U. Ebert, dr. J. Rademacher
- TU/e: Prof. dr. M.A. Peletier, dr. A. Muntean, dr. I.S. Pop
- UL: Prof. dr. A. Doelman, dr. V. Rottschäfer
- UT: Prof. dr. S.A. van Gils, dr. A. Zagaris
- UU: Prof. dr. O. Diekmann
- VUA: Prof. dr. G.J.B. van den Berg, prof. dr. J. Hulshof, dr. R. Planqué, dr. B. Rink

A number of participants within the theme Patterns and Waves were promoted to professor during the reporting period: Van Gils (UT), van den Berg (VU).

2.1 Description of the research theme

At present, mathematical techniques can only cover the most idealized behavior of complex systems. Many patterns formed in nature are induced by interacting instability mechanisms or competing localized structures [36][61]. The existing theory strongly exploits the underlying linear structures, but some patterns cannot be traced back to linear mechanisms [112]. The development of novel mathematical procedures by which these patterns can be studied is essential to the successful application of dynamical systems theory to the earth and life sciences. Such procedures will involve ideas from spatial dynamics, spectral theory, (global) bifurcation theory, etc. [24][46][85][99].

2.2 Developments of the research theme

Together with many international collaborators, the cluster members study pattern formation from a multitude of perspectives and in a variety of application areas, including diblock copolymers [48][30], lipid bilayers (cell membranes) [97], spiking neurons to brain waves [78], phytoplankton [115], tip growth in fungal hyphae [65], MEMS switches [108][5], twisted rods [54], combustion [14], lightning [44], Josephson junctions [35], and population movement under climate change [7].

Complementary, innovative developments in mathematical techniques are taking place. In particular, novel results are obtained in such areas as the elastica functional [86], periodic orbits in general Hamiltonian dynamics [15], localized structures in lattice dynamics [52], braided solutions in Cauchy-Rieman flows [13], and modulated wave trains [40]. It is remarkable that frequent exchanges of ideas occur between the finite- and infinite-dimensional setting: ordinary and partial differential equations often go hand in hand.

The envisaged mathematical techniques of spatial dynamics, spectral theory, and bifurcation theory have indeed been successfully applied to a number of problems in the earth end life sciences [53]. Furthermore, asymptotic expansion techniques and matched asymptotics have been extensively used, both rigorously and formally [3]. Progress had been made in the validation of the role of Kuramoto-Sivashinsky equation in describing cellular instabilities in planar fronts for various free boundary problems [22].

A major new direction of research has been weak descriptions of pattern types. In recent years a large variety of such weak characterizations has been uncovered, especially in the study of complex systems [2][51][1]. In most cases this has been possible due to the availability of a variational formulation.

More generally, variational techniques have been used to obtain sweepingly general results on global pattern formation phenomena in two dimensional maps, fourth order differential equations, and nonlinear Cauchy-Riemann PDEs [8][50]. In the future plans of the cluster we include a new theme *Variational Methods* (see Part C below).

2.3. Most important research highlight of the theme

When heating from below a fluid between two plates, a finite-wavelength instability leads to the occurrence of so-called Rayleigh-Bénard convection rolls. The Swift-Hohenberg equation is the principal model equation for this system [109]. We have proved the existence of a stationary periodic pattern with prescribed geometric properties. This single periodic pattern implies the nontriviality of a topological index, hence the existence of an abundance of complicated stationary solutions. Taking advantage of the associated variational framework, we can prove that all these solutions are stable under physical perturbations. We are thus able to draw conclusions about the complexity of the fluid dynamics. In particular we infer that when the temperature difference in a Rayleigh-Bénard cell is sufficiently large, then the Swift-Hohenberg model predicts that there are many different attractors, all corresponding to geometrically different velocity profiles.

- **Berg, J.B. van den**, J.-P. Lessard, K. Mischaikow, *Global smooth solution curves using rigorous branch following*, to appear in Mathematics of Computation, 2010

2.4. Five key publications

- Berestycki, H., **O. Diekmann**, C.J. Nagelkerke, **P.A. Zegeling**, *Can a species keep pace with a shifting climate?* Bull. Math. Biol. 71, no. 2, pp. 399-429, 2009
- Derks, A**., A. Doelman, S.A. van Gils**, H. Susanto, *Stability analysis of kinks in a Josephson junction*. SIAM Journal on Applied Dynamical Systems (SIADS), 6 (1), pp. 99-141, 2007
- **Doelman, A.,** B. Sandstede, A. Scheel, G. Schneider, *The dynamics of modulated wave trains*, Memoirs of the AMS 199 (934), 2009
- **Duijn, C.J. van**, L.A. Peletier, **I.S. Pop**, *A new class of entropy solutions of the Buckley-Leverett equation*, SIAM J. Math. Anal. Vol. 39, pp. 507-536, 2007
- **Peletier, M.A., M. Röger**, *Partial Localization, Lipid Bilayers, and the Elastica Functional* (math-ph/0607024), Archive for Rational Mechanics and Analysis, 193, pp. 475-537, online at http://dx.doi.org/10.1007/s00205-008-0150-4, 2009

Theme: Statistics

Introduction

Statistics was integrated in NDNS+ with the aim of joining all mathematical research directed at applications of mathematics in the life sciences. It was recognized that in many applications an interplay of analysis and stochastics is important, and that traditional boundaries in mathematics were confusing to life scientists.

1. People

- RuG: Prof. dr. E. Wit, prof. dr. E. Verbitskiy
- TU/e: Prof. dr. J.H. van Zanten (from 2009)
- UL: Prof. dr. R.D. Gill, prof. dr. J.J. Meulman
- VUA: Prof. dr. M.C.M. de Gunst, prof. dr. ir. G. Jongbloed, prof. dr. A.W. van der Vaart, prof. dr. J.H. van Zanten (until 2009), dr. F. Bijma, dr. M.A. Jonker, dr. M.A. van der Wiel, dr. W. van Wieringen

During the reporting period Wit, Verbitskiy, and Van Zanten were promoted to full professor.

2.1. Description of the research theme

The Life Sciences were named the 'science of the 21th century'. The first few years of this revolution were dominated by 'genomics', triggered by the complete sequencing of the genomes of humans and other organisms, and by the advent of new experimental technologies that produce massive amounts of data (e.g. microarrays, mass spectronomy, SNPs, imaging, cortical recording techniques). Because probability models explicitly allow for indeterminism, they are highly suitable for the modelling of complicated processes in living cells, as well as the behaviour of organisms and populations that are their aggregates. In addition to modelling there are many opportunities and needs for statisticians helping to analyse data arising from the new experimental platforms. Such data is typically very high-dimensional, with a relatively low signal-to-noise ratio. Many traditional statistical techniques are not well suited to this situation, and may lead to false positive results. New techniques must take account of the large amount of noise and variation, the high dimensionality of the underlying phenomena, as well as computational possibilities. Relevant themes are biological networks (neuronal networks, gene networks), medical imaging (PET, MEG, fMRI), chromosomal aberrations in cancer (CGH), gene expression, and statistical genetics (twin research, complex diseases, survival data).

2.2. Development of the research theme

The research has covered a wide range of subjects, both theoretical and applied. The theory concerns mostly statistical models of high or infinite dimension, with as main themes Bayesian methods, inverse problems, high-dimensional parameters, clustering and regression for high-dimensional data, multiple testing procedures, and statistics for stochastic processes. The 'applied work' was carried out in joint projects together with subject-area experts in the life sciences, mostly at UL and VUA and their medical centres. The main application areas include genomics (expression and CGH arrays, proteomics), metabolomics, medical imaging (PET, MEG, fMRI), genetics (twin studies, linkage, association), cell biology (ion channels, regeneration), neuro science (genomics, spike trains, transcription regulatory networks), carcinogenesis, forensic statistics, drug design, and medical statistics (survival analysis, breast cancer). While these projects are strongly problem- and data-driven, in many cases methods were developed with a wider applicability, or applications were viewed as examples of a general structure. The mathematical study of these examples and methods has highlighted fundamental issues. Invariably the applications concern very high-dimensional phenomena (such as 100 000 SNPs, multidimensional projections of noisy brain signals, etc.). Our contributions concern both new methods to handle such phenomena and theory that validates these methods and investigates their precision relative to potential other methods.

The totality of activities is in line with the original plan. We are particularly happy with the great number of truly multidisciplinary projects that were started. In our original plans theory and practice were expected to be closely linked, but we have found that the full potential of this interaction will take more time to develop. Regarding the integration of analysis and stochastics, we are very happy with what has been achieved, but feel that also here more time is needed. For instance, regarding population dynamics and systems biology more is to follow.

2.3. Most important research highlight of the theme

A variety of electrical rhythms, on different spatial and temporal scales, can be recorded in the mammalian nervous system. Coordination between these rhythms is thought to be one of the mechanisms by which functional units in the brain coordinate their activity. In particular, spatially separated neuronal entities synchronize by adjusting the frequency and/or phases of their rhythms via excitatory synaptic projections. To study these mechanisms we fitted coupled stochastic dynamical systems to recordings of MEG in humans and to recordings at electrodes placed in prepared mouse brain slices. Because these measurements are extremely noisy, appropriate statistical techniques are essential, where in the human measurements we also had to deal with the fact that the measurements are projections of magnetic activity deeper in the brain, leading for instance to volume conduction. We fitted both discrete time models, and continuous time Markov diffusion models. A first paper is to appear in Human Brain Mapping in 2010.

2.4. Five key publications

- **Bijma, F.,** J.C. de Munck, *A space-frequency analysis of MEG background processes,* NeuroImage, 43(3), pp. 478-488, 2008
- **Jonker, M.A., S. Bhulai**, D.I. Boomsma, R S.L. Ligthart, D. Posthuma, **A.W. van der Vaart,** *Gamma frailty model for linkage analysis with application to interval-censored migraine data*, Biostatistics, 10(1), pp. 187-200, 2009
- Rigat, F., **M. de Gunst**, J. van Pelt, *Bayesian modelling and analysis of spatio-temporal neuronal networks*, Bayesian Anal., 1(4), pp. 733-764 (electronic), 2006
- Vaart, W. van der, **J.H. van Zanten**, *Adaptive Bayesian estimation using a Gaussian random field with inverse gamma bandwidth*, Ann. Statist.,37(5B), pp. 2655-2675, 2009
- Wieringen, W.N. van, M.A. van de Wiel, **A.W. van der Vaart**, *A test for partial differential expression*, Journal of the American Statistical Association, 103(483), pp. 1039-1049, 2008

National and international collaboration for all themes

A large number of new connections have been established during the evaluation period. Some notable new collaborations at the national level are:

- The formation of an Applied Mathematics Institute among the technical universities (UT, TU/e, TUD), under director Geurts (UT)
- A strong VU-TU/e link, with multiple PhD projects co-supervised between the two institutes
- A strong CWI-UT link, with Verwer (CWI) and Botchev (UT) collaborating on Maxwell equations, and Frank (CWI) and Bokhove (UT) on atmospheric flows.
- A structural UL-CWI-UT collaboration by the joint projects shared between between Doelman, Rademacher and Zagaris on the mathematical analysis of pattern formation in ecological problems (deserts/vegetation patterns and phytoplankton).

Examples of national collaboration with non-mathematicians, in addition to those given in Section 7 of part A, are:

- Various groups have become members of national or local institutes with a primary focus outside mathematics: VU and UL in the Center for Medical Systems Biology (CMSB); CWI and VU in the Netherlands Institute for Systems Biology (NISB); VU in the Neuro Campus Amsterdam.
- RuG has developed joint activities with partners from bioengineering, biology and the medical sciences at RuG on the topic of systems and synthetic biology and networked system dynamics.
- The TU/e group has co-founded the interdisciplinary Institute for Complex Molecular Systems, and is involved in the Eindhoven Multiscale Institute (in preparation).
- The statistical groups of VUA, UL and RUG are developing a master program in Statistical Science with the statistics groups at Wageningen and Rotterdam medical center. They also have started numerous cooperations with scientists from the Leiden/AMsterdam Centre for Drug Design (LACDR), the Cancer Center Amsterdam (CCA), the micro-array facilities at LUMC and VUMC, the departments of Biological Psychology, Molecular and Cellular Neurobiology, Medical Genomics, Molecular Pathology, Pathology, Neurophysiology at VUA, and the Netherlands Institute for Neuroscience.

There are also many new international connections. Below follow just a few examples:

- Stevenson (UvA) is a partner in DFG-Schwerpunktprogramm 1114 addressing wavelet methods for time series and image processing.
- Geurts (UT) is chair of an EU COST Action LESAID (2006-2010) whose target is to form best practice advice for large-eddy simulation.
- Ebert was Dutch representative in the MT of EU-COST action P18 on Lightning (2004-2009), and is Dutch representative in the MT of the European research network (GDRE) E-CANES (Electrodynamic Coupling of the Atmosphere with Near-Earth Space, 2008-2013).
- An international collaboration was set up between Leiden and the research group of Sterk (AMC) with the group of Frey, University Hospital, Inselspital, Bern, Switzerland as part of an European project on ‘Unbiased Biomarkers for the Prediction of Respiratory Disease Outcomes’.
- Broer (RuG) has collaborations with Ciocci (PhD student RUGhent), Vanderbauwhede (RU Ghent) and Sevryuk (Russian Academy Moscow) on quasi-periodic bifurcation theory, with Cushman (UU, Calgary), Fassò (Pisa) and D.A. Sadovskií (Dunkerque) on global KAM Theory, with Simò (UBarcelona) and Vitolo (Exeter) on the mildly degenerate Hopf-Neimark-Sacker bifurcation.
- Life-science modelling is being carried out by the RuG and UU groups in cooperation with Poggiale (UMarseille), Roussarie (Dijon), Berestycki (Chicago). The CWI Life Sciences group collaborates with St. Petersburg State Polytechnical University and Ioffe Institute (RU). Frank and Crommelin (CWI) have intensified contacts with Leimkuhler (University of Edinburgh) and Reich (Imperial College).
- Doelman intensified contacts with Beck and Kaper (Boston), Derks (Surrey), Nishiura (Sapporo), Promislow (Michigan and Kloostermanhoogleraar 2010), Sandstede (Brown), Scheel (Minneapolis)l, and Susanto (Nottingham).

- The existing connections with Brown University (Mallet-Paret, Sandstede) were reinforced by the two Rubicon fellowships to Brown (Hupkes, Van Heijster).
- The VU analysis group has further extended its collaborations with UPenn (Ghrist), Rutgers (Lessard, Mischaikow), and Vancouver (Williams), as well as Angenent (Wisconsin), Bertsch (IAC-Rome), Sivashinsky (Tel Aviv), Lorenzi (Parma), and Brauner (Bordeaux).
- Peletier gave a series of Lipschitz Lectures at Universität Bonn on Variational Methods for the Analysis of Patterns.
- The TU/e group has created new ties with Bonn (Otto, Müller, Conti), Dortmund (Schweizer, Roeger, Veneroni), Erlangen (Knabner), Stuttgart (Rohde, Helmig, Niessner), Jena (Attinger, Radu), Paris (Cancés, Vohralik), Rome (Garroni, Braides), and Vancouver (Williams, Choksi).
- Ebert co-supervises Daria Dubrovin, PhD student with Yoav Yair (Open University Israel) and Colin Price (Tel Aviv University), for experimental and simulation work on lightning phenomena on other planets performed at CWI and TU/e. The results will be used for designing international satellite and space missions.
- Ebert furthermore collaborates through joint publications with B. Davidovitch (U Massachusetts), D. Dubrovin (Tel Aviv), G. Derks (U Surrey), Y. Goto (Tohoku Gakuin U, Tagajo), I. Grekhov (St. Petersburg), C.-Y. Kao (U Ohio), A. Minarsky (St. Petersburg), S. Pancheshnyi (Toulouse), C. Price (Tel Aviv), P. Rodin (St. Petersburg), L. Schäfer (Essen-Duisburg), D. Sentman (U Alaska, Fairbanks), S. Tanveer (U Ohio), Y. Takahashi (Tohoku U, Sendai), Y. Yair (Tel Aviv), R. Yaniv (Tel Aviv).

Other scientific activities, for all themes combined

A central element of the NDNS$^+$ programme was the organisation, support, and facilitating of work-shops. Below is a list of workshops organized by NDNS$^+$ members, all with more or less support from NDNS$^+$. Many were held in the Lorentz Center in Leiden.

Lorentz Center wokshops:

- *The multiscale nature of spark precursors and high altitude lightning*, May 9 – 13, 2005, organisers: **U. Ebert**, M. Füllekrug, P.F. Williams
- *Dynamics of Patterns*, November 7 – 11, 2005, organisers: **O. Diekmann, A. Doelman, J. Hulshof**, B. Mulder, W. van de Water, W. van Saarloos
- *Limit problems in Analysis*, May 1 – 5, 2006, organisers**: J.B. van den Berg, J.F. Williams**
- *Computational and topological aspects of dynamics,* May 15 – 19, 2006, organisers: **J.B. van den Berg, R.C.A.M. van der Vorst**
- *Geometric and Multiscale Methods for Geophysical Fluid Dynamics*, October 2 – 6, 2006, organisers: **J. Frank, O. Bokhove**
- *Mathematical Modeling and Analysis of Biological Networks,* NDNS+ workshop, January 29 - February 2, 2007, organisers: **S.M. Verduyn Lunel, A.W. van der Vaart**
- *Nonlinear Collective Behaviour: Networks, Swarming and Reaction Diffusion Dynamics,* June 11 – 15, 2007, organisers: R. Kapral, K. Showalter, **A. Doelman**
- *Streamers, sprites, leaders, lightning: from micro- to macroscales*, October 8 – 12, 2007, organisers: **U. Ebert**, D.D. Sentman
- *Hamiltonian Lattice Dynamical Systems,* October 15 – 19, 2007, organisers: **B.W. Rink**, J.S.W. Lamb
- *Numerical Modelling of Complex Dynamical Systems,* May 6 – 9, 2008, organisers: **J. Blom, J. Frank, W. Hundsdorfer, B. Sommeijer, J. Verwer**
- *Operator Structures and Dynamical Systems,* July 21 – 25, 2008, organisers: **M.F.E. de Jeu**, N.P. Landsman, S. Silvestrov, C.F. Skau, J. Tomiyama, **S.M. Verduyn Lunel**
- *PDE approximations in Fast reaction - Slow diffusion scenarios,* November 10 – 14, 2008, organisers: T. Aiki, D. Hilhorst, M. Mimura, **A. Muntean**
- *KAM Theory and its applications*, December 1 – 5, 2008, organisers: **H.W. Broer, H. Hanßmann**, M.B. Sevryuk

- *Mathematical challenges in climate science,* March 9 – 13, 2009, organisers: **D. Crommelin**, P.J. van Leeuwen, R. Kuske
- *Experimental design in systems biology Data analysis and parameter identification,* June 2 – 5, 2009, organisers**: J. Blom,** A.K. Smilde
- *Monodromy and geometric phases in classical and quantum mechanics*, June 15 – 19, 2009, organisers: **K. Efstathiou**, J. Robbins, D. Sadovskii, **H. Waalkens**
- *Brain Waves*, June 22 – 26, 2009, organisers: C.C.A.M. Gielen, **S.A. van Gils**, M.J.A.M. van Putten, D. Terman
- *Metabolic Pathways Analysis 2009,* October 26 – 30, 2009, organisers: **F. Bruggeman**, B. Olivier, M.F. Sagot, S. Schuster, L. Stougie
- *Coherent structures in evolutionary equations,* July 12 – 16, 2010, organisers: G. Pavliotis**, J.D.M. Rademacher**
- *Symplectic techniques in conservative dynamics,* August 2 – 6, 2010, organisers: V. Ginzburg, **F. Pasquotto, B. Rink, R.C.A.M. Van der Vorst**
- *Poly and Polymer Electrolytes for Energy Conversion: Ab Initio, Molecular, and Continuum Models*, August 23 – 27, 2010, organisers: **A. Doelman**, K. Promislow, S. Paddison
- *4th Euro-Japanese Workshop on Blow-up,* September 6 – 10, 2010, organisers: M. Fila, **J. Hulshof**, J.L. Vazquez, E. Yanagida
- *Modelling angiogenesis: joining cells, maths and computers,* October 4 – 8, 2010, organisers: **R.M.H. Merks**, E. Giraudo, P. Koolwijk, B. Ribba

Other workshops and conferences:

- *52nd European Study Group Mathematics with Industry*, January - February 2005 (**Van den Berg**, Bhulai, **Hulshof**, Koole, Quant, Williams)
- *55th European Study Group Mathematics with Industry*, January - February 2006 (Fledderus, Van der Hofstad, Jochemz, **Molenaar**, Mussche, **Peletier**, **Prokert**)
- *58th European Study Group Mathematics With Industry*, January - February 2007 (Bisseling, Dajani, Dijkema, Van de Leur, **Zegeling**)
- *63rd European Study Group Mathematics with Industry*, January - February 2008 (**Bokhove**, Hurink, Meinsma, **Stolk**, Vellekoop)
- *Oberwolfach mini-workshop 'Mathematics of biological membranes*', September 2008 (**Peletier**, Röger, Garcke, Niethammer)
- *72nd European Study Group Mathematics with Industry*, January 2010 (Van der Mei, **Frank**, Bouman, den Boer, Bosman, Verhoef)
- *KNAW Academy Colloquium 'Multi-scale problems in sustainable resource management'*, September 2010 (**Pop**, **Peletier**)

Interrelationships between mathematics and the Life Sciences

The creation of several permanent and temporary positions five years ago, both in nonlinear dynamics for life sciences and in stochastics for life sciences, enlarged the size of the small Dutch biomathematics community at exactly the right time. All over the world mathematicians were ready to face the challenge posed by new developments in life sciences and their technology. Also in the Netherlands, life-scientists were actively turning to mathematicians for help, stretching the capacity of the Dutch biomathematics community. Thanks to the cluster there was a larger pool of mathematicians who could react to this demand.

The cluster was not only directly advantageous for the size of the biomathematics community, but it also indirectly stimulated further community-building. A relatively large number of proposals for PhD and postdoc positions in the area of biomathematics were funded. Moreover, the cluster has brought researchers from nonlinear dynamics and stochastics together, not only via workshops—several of the NDNS$^+$-workshops specifically aimed for a mixed audience in that respect—but also via the 'advisors', initiatives such as the NDNS$^+$ Biomathematics seminar in Leiden, and within the respective mathematics institutes themselves. The two groups of researchers have become acquainted and, as a result, more open to each other's methods and ideas. They have started to explore the potential for collaboration both small-scale and in larger-scale projects. Here we are only at the beginning, but thanks to the cluster the foundation has been laid. It is expected that in the next decade this will be developed further, in particular within the context of the application areas systems biology and the neurosciences.

Part of the success of the Life-Sciences connection is visible in a very concrete way in new programmes and groups. The NDNS$^+$ Workshop 'Control Theory for Systems Biology' (Groningen, October 2007) brought together participants from bio-engineering, biology, biochemistry, systems and control theory, and the medical sciences. It also paved the way for the PhD Summer school Cells and Systems (Woudschoten, June 2008), organized by the Dutch Institute of Systems and Control (main organizer A.J. van der Schaft) in collaboration with the Kluyver Centre for Genomics of Industrial Fermentation (co-organizer B. Teusink). This activity of NDNS$^+$ in the area of systems biology was further stimulated by the NWO funding of the Systems Biology Centre for Energy Metabolism and Ageing at the University of Groningen (including NDNS$^+$ partners E.C. Wit and A.J. van der Schaft) in the beginning of 2010.

At the CWI, Doelman and Verwer played instrumental roles in the founding of a Life Sciences research group at CWI in 2008. Two tenure-track positions have been opened and filled (Gunnar Klau, Roeland Merks), and a third is in preparation, as are a number of PhD and postdoc positions. The CWI group cooperates with the Netherlands Cancer Institute (NKI) and the French INRIA, and CWI is one of the four partners of the Netherlands Institute for Systems Biology (NISB). The existing activities and collaborations within NDNS$^+$ were instrumental in providing a strong foundation for these further Life Sciences activities.

Key publications:

- **Hille, S.C**., *Local well-posedness of kinetic chemotaxis models*, J. Evol. Equ. 8, no. 3, pp. 423-448, 2008
- Hoekstra, M., M. Vogelzang, **E. Verbitskiy**, M.W.N. Nijsten, *Health technology assessment review: Computerized glucose regulation in the intensive care unit – How to create artificial control?,* Critical Care 13:223, 2009
- Draisma, H.M., Th.H. Reijmers, I. Bobeldijk-Pastorova, **J.J. Meulman,** et al., *Similarities and differences in lipidomics profiles among healthymonozygotic twin pairs,* Omics, A journal of integrative biology, 12, pp. 17-31, 2008
- Sendova-Franks, A.B., R.K. Hayward, B. Wulf, T. Klimek, R. James, **R. Planqué**, N.F. Britton, and N.R. Franks, *Emergency networking: famine relief in ant colonies*, Anim. Behav. doi:10.1016/j.anbehav.2009.11.035, 2009

Interrelationships between mathematics and the Earth Sciences

In the original proposal, the essence of the relation, or cross-fertilization, between the $NDNS^+$ cluster and the earth sciences was formulated as follows:

Although the models of earth sciences are in general composed of many strongly coupled highly nonlinear partial differential equations that act on various spatial and temporal scales, the dynamics generated by these models are often surprisingly simple, especially on finite, but long, intervals of time. Moreover, the complex phenomena exhibited by these models can most often be seen as the products of interaction between relatively simple 'building blocks'. In the terminology of dynamical systems, this complex behavior might thus be seen as a bifurcation scenario that is initiated by the destabilization of a fundamental pattern.

The most urgent challenge posed by the earth sciences to the field of nonlinear systems is to develop methods by which this low-dimensional behavior of the infinite-dimensional complex multi-scale models can be derived, understood and predicted.

Apart from the theme 'Networks and Delays', which has a strong focus on the life sciences, this challenge permeates all themes of the $NDNS^+$ cluster, and in fact unites them. Without exaggeration, one can say that the cluster has been very successful in its response to the challenge formulated above. We illustrate this with an example for each theme.

Bifurcations and chaos. Together with researchers from the Institute for Marine and Atmospheric Research Utrecht (IMAU, UU), especially Henk Dijkstra, the group of Henk Broer at Groningen University has worked on the recognizing low dimensional behavior in reduced models for atmospheric/oceanographic flows – in the sense of the celebrated Lorenz model (but with approximately 50 instead of 3 modes) – as is described above in the 'Most important research highlight of the theme *Bifurcations and Chaos'*.

Scientific computing. At the CWI, and especially in the group of Jason Frank, numerical methods and algorithms have been developed that conserve essential properties of the flows generated by PDE models, such as conserved quantities and symmetries. The methods from this field of geometric numerical integration are extremely relevant in the context of long term weather simulations, and thus in climate dynamics. Moreover, they are very suitable for the following the evolution of low-dimensional localized structures such as vortices in the atmosphere (e.g. hurricanes) and the ocean (eddies). There are close research contacts between this group and researchers at the KNMI (Royal Netherlands Meteorological Institute) and the NIOZ (Royal Netherlands Institute for Sea Research), as well as UT, TUD, the Courant Insitutute, University of Edinburgh, and Universität Potsdam.

Transient dynamics and multiple scales. Another group at the CWI, the group of Ute Ebert, is an internationally leading research group on modeling, simulating and analyzing streamer discharges and, consecutively, a variety of phenomena in lightning and high-voltage and plasma technology. As described in 'Most important research highlight of the theme *Transient Dynamics and Multiple Scales'* above, their recent work provides essential novel insights in understanding localized atmospheric structures such as sprites and halo's.

Patterns and waves. Inspired by [64] a research project has been initiated by the group of Arjen Doelman (now in Leiden) and Huib de Swart (IMAU, UU) on the interactions between (localized) phytoplankton blooms and the hydrodynamics of oceans and/or of estuaries. This project is still at an early stage, but it has already generated novel insights in the nature of (localized) phytoplankton blooms, also called 'deep chlorophyll maxima'. The project has strongly profited from the formation of the cluster: it has been initiated through the weekly contacts between de Swart and Doelman (de Swart had a one day a week position at the CWI financed by the cluster). Moreover, significant parts of the positions of the two junior scientists involved in this project, Julia Zijlstra (PhD student) and

Antonios Zagaris (postdoc at the CWI and the University of Amsterdam, now tenure track in Twente) have been partially funded by the cluster.

In fact, the success of the research within the cluster in the field of earth sciences formed (for a part) the motivation to define a new theme of research within the cluster for the upcoming years, the theme **Stochastic dynamics** (see Part C below). The work of Daan Crommelin, whose tenure-track position has been created by the cluster, focuses on the interactions between the earth sciences (especially meteorology and climate), dynamical systems, and stochastic processes. One of the main themes of his work is determining low-dimensional (stochastic) models for regime behavior in high-dimensional descriptions of atmospheric flows.

Through the formation of the cluster, one can indeed say that the already existing interactions between 'nonlinear mathematicians' and earth scientists received the desired impetus towards the next level that goes beyond that of individual contacts.

Key publications:

- **Dubinkina, S., J. Frank**, *Statistical mechanics of Arakawa's discretizations*, J. Comput. Phys. 227, no. 2, pp. 1286-1305, 2007
- **Luque, A., U. Ebert**, *Emergence of sprite streamers from screening-ionization waves in the lower ionosphere*, Nature Geoscience 2, pp. 757-760, 2009
- Majda, A.J., C.L. Franzke, A. Fischer, **D.T. Crommelin**, *Distinct metastable atmospheric regimes despite nearly Gaussian statistics: a paradigm model*, Proc. Natl. Acad. Sci. USA 103, no. 22, pp. 8309-8314, 2006
- **Sterk, A.E., R. Vitolo, H.W. Broer**, C. Simò, **H.A. Dijkstra**, *New nonlinear mechanisms of mid-latitude atmospheric low-frequency variability*, Physica D: Nonlinear Phenomena 239, pp 701-718, 2010
- **Zagaris A, A. Doelman, N.N.P. Thi, B.P. Sommeijer,** *Blooming in a nonlocal, coupled phytoplankton-nutrient model*. Siam Journal of Applied Mathematics 69 (4), pp. 1174-1204, 2008

Part C: New cluster plans

1. Research plan

NDNS$^+$ was created, five years ago, to address the inherent complexity of natural systems, and of the mathematical models and systems that we use to describe them. While important advances have been made – see the earlier part of this report – the challenges that the natural world presents to science are as large as ever. The research themes that we formulate below for the coming five years reflect the significant challenges that face us, and the opportunities that we now have to address these.

Some of the research themes of this programme are organized around tool sets for the analysis of differential equations. *Bifurcation theory, Variational Methods,* and *Scientific Computing* are three complementary sets of tools for the analysis of differential equations, and in this programme we develop these three areas further to meet the specific demands of the complex systems that arise in the Earth and Life Sciences. In addition, the cluster provides a unique opportunity to bring these areas together and enhance their interaction.

Three other themes are oriented more towards phenomena than tools. A common element of complex systems in general, and specifically in those that arise in the natural environment, is their *Multi-scale* nature: behaviour that is well described at one scale is intrinsically dependent on behaviour at other (larger or smaller, faster or slower) scales. The understanding of *Patterns* and their evolution is central in the study of complex natural systems. Finally, the role of stochastic effects in otherwise deterministic models has strongly increased in the last five years, and we will develop this issue in the new theme *Stochastic Dynamics.*

Of course the division into separate themes is only a choice of labelling, and many activities of the cluster will be part of several themes at the same time (see Section 1.2 below).

1.1. The different research themes

Bifurcations and chaos:

In the theme *Bifurcations and Chaos* we will continue a number of successful activities in collaboration with (inter-) national networks as described above. First of all this concerns the climate research in collaboration with Simò (Barcelona), Sterk, and Vitolo (Exeter) [106], where now also statistics of extreme events is included in the research plans. Secondly, we will continue the fundamental research in the direction of quasi-periodic bifurcations, in collaboration with Hanßmann (UU) and Wagener (UvA) [23], and the Hopf Saddle-Node bifurcation for maps, again with Simò and Vitolo [27]. The global theory of integrable and nearly integrable systems will be extended in directions of differential geometry and algebraic topology that will lead to a better understanding of various versions of monodromy and quantum monodromy; this will be in collaboration with Efstathiou, Tangerman (Stony Brook), Waalkens, and others. A future life-science project deals with subjects from chronobiology, in particular with models for pace maker cells in terms of periodic (averaged) or multiperiodic oscillators, in collaboration with Beersma. Moreover, the study of global bifurcations of (networks) of connecting orbits is flourishing [63][62].

Recent work on chaotic dynamical systems, in particular partially hyperbolic dynamics, and random dynamic systems, have revealed deep connections between them, and have opened ways to further our understanding of both of them. This is internationally a very active area of research, in which we intend to keep contributing [117]. The research can be expected to increase our understanding of important effects such as synchronization and clustering that are frequently observed in large-scale systems. There are obvious connections with the theme *Stochastic Dynamics.*

Dynamics on networks, also large networks, may show effects such as switching and intermittency. Research will be focussed on an understanding of these phenomena. Global bifurcation theory plays an essential role in this study [62].

Multiple scales:

Many systems in applications show complex behaviour that arises from effects at different spatial and temporal scales. Analysis of these systems requires a subtle managing of both the modelling at each separate scale and the propagation of information between the scales. A particular difficulty is the handling of fluctuations and oscillations, and their consequences at larger scales via nonlinear interactions. This sub-area is especially fertile, since there is an intimate relationship between experimental observations, computer simulations, modeling, numerical analysis and dynamical systems theory [46][61][66].

Promising analytical directions in this field are for instance the identification of coherent structures that typically correspond to (evolving) heteroclinic networks [91] whose interactions can be understood in terms of reduced models [55][56]. However, this is at present only possible if these interactions are weak, or semi-strong, while the most intriguing behavior is exhibited by strongly interacting structures [83]. Another promising direction is the use of gradient-flow structures to enable scale transitions to handle the nonlinear interactions between fluctuations [104]. Research on the interaction between particle and continuum systems is growing in many fields, such as in simulation of complex fluids (hybrid particle-continuum methods [41][60] in agent-based models of large populations [77][57], dislocations in crystals [47], the continuum limit of coupled oscillators on lattices [96], and particle aspects in discharges [73].

The computational aspects of multiscale systems are important and challenging, and we discuss them below as part of the theme *Scientific Computing*. It is important to remark, however, that additional cross-fertilization arises between the theoretical and the numerical analysis of a multiscale system: analytic understanding leads to fine-tuned numerical methods, and effective numerical methods lead to analytic conjectures. In practice there will therefore be strong interaction between the theoretical and numerical approaches to multiscale systems.

Scientific computing:

Multiscale systems present a specific set of challenges to the field of scientific computing. On account of the multitude of scales in such problems, the computational complexity of conventional approximation methods is prohibitive, and new optimal-adaptive approximation techniques based on a-posteriori error estimates have to be developed [107]. For fixed computational complexity, such optimal-adaptive methods minimize the error, either in a suitable norm or in a particular functional, depending on the goal of the computation. *Adaptivity* can pertain to both the numerical approximation and the underlying mathematical-physical model. In the latter case (i.e. *model adaptivity*), the adaptive process can for instance select between a micro-scale heterogeneous model and a locally homogenized approximation, between a molecular model and a corresponding continuum approximation, or between a high-dimensional system and a low-dimensional approximation of that system, depending on the contribution of local errors to the object of interest [105]. A fundamental challenge concerns adaptivity between disparate models, e.g., between models of distinct dimensionality, between a discrete molecular model and a corresponding continuum approximation, or between a model that is deterministic on one scale and stochastic on another. Because the adaptivity relies on precise a-posteriori error estimates, exact error relations between the different models have to be derived, which can for instance require detailed knowledge of hydrodynamic limits and entropy relations (transitional molecular/continuum models) or probability theory (transitional stochastic/deterministic models) [34]. Often an assumption of scale separation is needed and this may not be valid. In deriving these error relations, there is significant potential for collaborations and cross-fertilization with other branches of mathematics, notably, with applied analysis and dynamical-systems theory. In particular internal collaboration is expected with the NDNS$^+$ themes *Multiple Scales* and *Stochastic Dynamics*. Furthermore, despite the enormous progress that has been made in recent years, many open issues with regard to convergence and optimality of adaptive (in the sense of approximation) methods still remain, especially for nonlinear problems.

Research within this theme will address challenges such as the construction of numerical methods for multiscale phenomena, using stochastic subgrid scale models, synthesis of particle and continuum models, and computational methods for stochastic (partial) differential equations; space-time adaptive methods for 3D applications in fluid mechanics, electromagnetics, and plasmas, and adjoint techniques for error control; development of methods for high-dimensional PDEs – e.g. sparse-grid and Monte Carlo methods, and dimension-reduction methods for financial mathematics, and the Boltzmann equation; mathematical foundations for multi-scale methods – convergence of combined particle-continuum methods and in the absence of scale separation, convergence of adaptive finite element methods; developments of numerical methods for emerging computer platforms, and GPU computing; and robust and efficient solvers for large linear systems. Finally, there is a surge in the feedback of sophisticated numerical methods into computational dynamics, in particular computer assisted proofs and efficient topological verification of computations of invariant sets [33][4][67][9].

Patterns and waves:

The analysis of patterns is a central problem in the theory of complex systems, with relevance to a large number of natural systems, ranging from nano-templating to desertification and from neuronal networks to celestial dynamics.

The participating groups within NDNS+ have a strong track record in this field, and the work in this theme will focus on developing and extending the existing lines of research. For instance, existing close-to-bifurcation understanding will provide a stepping-stone for more fully-nonlinear investigations such as that of the Busse Balloon [39] and of nonlinear front interactions [55][56].. Topological arguments based on e.g. Conley index theory are not limited to small-parameter regimes and provide complementary methods of characterizing solution types [11], possibly assisted by rigorous numerics [10].

Recent new developments concern the deep interactions between the mathematical theory of the dynamics of spatially periodic patterns, or wave trains, with questions from ecology [103][101]. Especially in the context of desertification, for which the appearance of vegetation patterns is a so-called early warning signal, extremely relevant questions such as ‘How close is this given vegetated terrain to the catastrophic collapse to the desert state?’ are intimately related to the character of the dynamics within families of wave trains under slowly, either or not stochastically, varying conditions [69][36]. This is a completely novel subject for research in which Doelman and Rademacher work together with the ecologist Max Rietkerk.

Because of the development in one-dimensional systems the analysis of two- and higher-dimensional systems is coming within reach. The multi-scale technique of variational singular-limit analysis provides weak characterizations of patterns with only approximate symmetries [87]. Furthermore, in two dimensions studies of localized patterns [6] and two-dimensional traveling wave and vortex structures [18][65] are becoming feasible. An especially promising subtheme within the cluster is the study of defects within ordered, say striped, (two-dimensional) media, since in this context the variational approach of van den Berg, Peletier and others can be, or even should be, combined with the dynamical-systems point of view of Doelman, Rademachers and co-workers.

Variational Methods:

Many natural systems tend to optimize a certain crucial quantity, be it free energy, evolutionary fitness, stresses, or curvature. This goes back to the duality in the description of classical mechanics: Hamiltonian dynamics (omnipresent in natural systems with negligible dissipation) has a variational formulation in terms of critical points of the corresponding Lagrangian action. More generally, in a plethora of natural problems the solution can be interpreted as a critical point of an associated functional, enhancing the possibilities to study these systems effectively. The optimal configurations seen in nature are, for such systems, usually not close to homogeneous. Quite the opposite: they often exhibit large contrast, such as highly localized concentrations or large amplitude variations.

The Calculus of Variations (in which functional analysis, geometry, topology and PDE theory interact closely) forms a perfectly suited mathematical toolbox for variational problems, both to study the eventual equilibrium states, as well as the evolution towards these (gradient flows). The equilibria are generally studied using various Morse-theoretic approaches. Gradient flows have attracted strong interest in geometry (leading to the celebrated proof by Perelman of the Poincaré conjecture via the Ricci flow [89]), while new development are not limited to geometry but also contribute to further understanding in the natural sciences. Examples include parabolic geometric flows such as the harmonic map flows (nematic liquid crystals) [12], the Willmore flow (red blood cells), and various generalizations such as for rate-independent systems [81].

Variational techniques constitute one of those areas in mathematics where theoretical advances and applications in natural science move forward in unison. Specific subjects where the cluster members have strong expertise, and where we expect seminal progress in the next decade, include singularity formation in geometric flows [12], periodic and quasi-periodic Hamiltonian dynamics [49], advances in Morse theory (Conley index and Floer homology) [13], singular limits (Gamma-convergence) [87], [88], free boundary problems and Wasserstein metrics [90], partial localization [86], localized structures in conservative lattice dynamics [42], generalized gradient flows [1], variational inequalities [28], control and optimization of Hamiltonian systems [84], and the network modeling of conservation laws and Hamiltonian systems [29].

Stochastic Dynamics:

Over the last few years the role of stochastic effects in 'classically deterministic' models has increased significantly. Several developments contribute to this trend, among which the trend to smaller systems with stronger thermal fluctuations, the trend towards more complex systems, and a general confluence of analysis with probability theory.

A central open problem, raised by Kolmogorov, is that of stochastic stability. This decides whether a study of deterministic models is meaningful for models with small random effects. These matters have a direct relevance for natural systems. As an example, recent studies include the effect of noise on synchronization and clustering phenomena found in models for cell-cycle dynamics [20]. There further are close connections between chaotic dynamics and random dynamics; there is a common set of tools (ergodic theory) and results from one perspective often have counterparts from the other perspective.

An important problem that has attracted interest in the Dutch scientific community is on the development of tools to predict critical transitions in noisy dynamical systems [100]. The related phenomenon of metastability also implies the possibility of different dynamics on larger time scales. This is an area where the themes *Bifurcations and Chaos* and stochastic dynamics touch. Without models at hand, one finds oneself in the position that the construction of stochastic models from data is needed that describe the dynamics of observables [31]. This is one area where *Scientific Computing* and stochastic dynamics come together; another arises from spatial coupling of stochastic particle models and deterministic PDE's, based on knowing where fluctuations and particle aspects are important and where not [72].

Patterns may be influenced by random impulses due to thermal fluctuations, by the stochastic parameterization of small scales, or by noise in some control parameter of the system. A deterministic system may also be perturbed by biased stochastic impulsive changes in state at random times [71], as e.g. encountered in kinetic models for chemotaxis [45][59] or in structured population models, where time- and state-at-birth of newborn indiviudals are randomly drawn from given state-dependent probability distributions.

Modelling such random perturbations leads naturally to stochastic partial differential equations. Many of such systems have natural state spaces that do not fit well in the standard Hilbert space approach. Only recently the mathematical framework for more general state spaces has been devopeloped for random perturbations that are continuous in time [82]. The extension to stochastic perturbations that are not continuous in time, nonsymmetric in space or that have heavy tail distributions is still in a pioneering stage [93]. The presence of delayed feedback may be an additional complication [92]. The persistance of stability of the underlying deterministic dynamics under such general random perturbations is still largely unknown and an important and challenging problem for upcoming research. Tools for the related analysis of evolutionary equations in spaces of measures needs to be further developed, aiming at properties such as existence, structure and stability of invariant and ergodic measures [114], especially in relation to suitable perturbation theory.

1.2. Coherence

The strong interaction between the different research themes of this cluster can be recognized in various ways. The list of participants and their respective themes (see Section C.3.2 below) shows the extensive personal unions between the themes. In terms of the research topics themselves, there are many natural interactions, some of which were already mentioned in the descriptions above. We mention a few others: Stochastic effects have a direct relationship with variational structures [1]; variational methods are of use in bifurcation analysis [74], scientific computing [116], scale transitions [86], and pattern analysis [87]; bifurcation analysis provides techniques for the study of coherent structures [38] and lies at the heart of the study of stochastic influences on such structures.

The themes of Multiple Scales and Scientific Computing are special, in that they address concepts and issues that are of relevance to all the other themes (and to many research topics besides this cluster as well). We formulate them here as separate themes nonetheless, since concentrated research in these directions will contribute to fundamental methodology, which then can be applied in the form of an enabling technology to the other themes.

2. Quality of the research team

In a practical sense, the research of the cluster will take place at the associated departments of the universities and the CWI. To coordinate this activity, we have listed eleven people listed below, corresponding roughly to one person per department. Each person has been chosen for being a local leader in their respective institutes, which enables them to bring the wider circle of collaborators in the 'home' institutes into the collaborative context of the NDNS+ cluster. Importantly, those that are selected also have a track record of collaboration with scientists from other disciplines.

We have made the choice of including groups and a group leader from each university with a mathematical group in the NDNS+ domain. The national mathematical research environment can only stay healthy if it manages to attract and educate a sufficiently large and sufficiently highly qualified body of students. The budget cuts that have hit every university make it increasingly difficult to have excellent researchers as the teachers of the next generation of students. External funding tends to focus on PhDs and postdocs, and therefore does little to alleviate the problem at the level of lecturers and higher. By broadly supporting mathematics departments, we aim to contribute to high-quality teaching and therefore to a long-term healthy research environment.

Prof. dr. J.B. van den Berg (url: www.math.vu.nl/~janbouwe/) is professor of Mathematics (Differential Equations with Applications) at the VU University Amsterdam. His research interests include nonlinear partial differential equations and dynamical systems, often leading to pattern formation. Subjects include (formal) asymptotic techniques, (rigorous) computational methods, the harmonic map heat flow (and other geometric flows), Hamiltonian systems, biomathematics, periodically modulated traveling waves, topologically validated computations, variational methods, Conley index, and Floer homology.
Van den Berg is director of education for mathematics and, besides several research grants, has secured the funding of the Study Groups Mathematics with Industry in the Netherlands for five years. He has supervised four PhD students and has published more than 40 papers.

Prof. dr. H.W. Broer (url: www.math.rug.nl/~broer/) is professor of Mathematics (Dynamical Systems, Geometry) at Groningen University and member of the Royal Netherlands Academy of Arts and Sciences (KNAW). His main research area lies in dynamical systems with an emphasis on the transitions between simple and more complicated systems, both from the fundamental and the applied point of view.
Broer is Editor-in-Chief of the KNAW journal Indagationes Mathematicae, the journal Discrete and Continuous Dynamical Systems and editor of publisher Epsilon Uitgaven (books). He has co-edited 7 books and (co)-authored 11 books, 75 refereed journal papers and 30 refereed contributions to proceedings. Moreover he has been PhD advisor of 23 students. His current h-index is 27 (Google scholar).
Broer is a member of a number of organizing- and program committees of international conferences and also is an invited speaker on many others. Broer is the scientific director of the Johann Bernoulli Institute for Mathematics and Computer Science of Groningen University, and he chairs the board of the Mathematics Research Institute (MRI) and the advisory council of the Groningen Graduate School of Sciences (GGSS).

Prof. dr. ir. E.H. van Brummelen is leader of the section Multiscale Engineering Fluid Dynamics in the department of Mechanical Engineering, and he holds a part-time position in the Mathematics department, both at TU/e. The common denominator of his research over the past years is the development, analysis, and application of advanced numerical methods for multiscale flow problems, with special empahasis on free-boundary and interface problems and flow problems in the transitional molecular/continuum regime. Recent work in the group of Van Brummelen has focussed on dual-based goal-oriented error estimation and adaptivity for free-boundary problems, coupled problems and the Bolzmann equation. Van Brummelen has received a Veni grant from the Dutch Science Foundation NWO in the Exact Sciences council area. Moreover, he is a recipient of the Bill Morton prize for young researchers in computational fluid dynamics. Professor van Brummelen is a board member of the Association of Innovational Research Grant Laureates. He has (co-)authored 19 papers and 2 book contributions. His current h-index is 10 (Google scholar).

Prof. dr. O. Diekmann (url: www.math.uu.nl/staff/Info/diekmann.html) is professor of mathematics (Applied Analysis) at Utrecht University. His main research area is mathematical population dynamics and epidemiology of infectious diseases. In this context his interests focus on dynamical systems, in particular (adjoint) semi-groups of operators acting on infinite dimensional state spaces (which also arise very naturally in the study of delay equations).
Diekmann is Honorary Editor of the Journal of Mathematical Biology and a former Associate Editor of the Japan Journal of Industrial and Applied Mathematics. He is currently Associate Editor of the Canadian Applied Mathematics Quarterly, Mathematical Models and Methods in Applied Sciences, Mathematical Biosciences, Theoretical Population Biology, International Journal of Biomathematics, and the Wiley Book Series Mathematical and Computational Biology. He has (co-)edited 3 books and (co-)authored 2 books, 77 refereed journal papers, 1 letter, 27 refereed contributions to proceedings, and 15 chapters in books. His current h-index is 37 (Google scholar).

Prof. dr. A. Doelman (url: www.math.leidenuniv.nl/~doelman/) is professor of 'Applied Analysis and Dynamical Systems' at the Mathematical Institute of Leiden University and Director of the Lorentz Center, an international center for workshops in the sciences (see http://lorentzcenter.nl). His main mathematical research interests are in the fields of pattern formation – from the theory of modulation equations to far-from-equilibrium dynamics – nonlinear dynamical systems, and singular perturbation theory. He is active in application areas such as oceanography, geophysics, chemistry, hydrodynamics, ecology, biology, etc. and has intensely collaborated with scientists in these fields. Doelman is Editor-in-Chief of Physica D (Nonlinear Phenomena), editor of the Journal of Computational Science, and (co-)organizer of numerous workshops and conferences. He has been (co-)coordinator the NWO-Nonlinear Systems platform '*Pattern Formation in the Natural Environment*', is one of the initiators/coordinators of the FOM-NWO program '*Dynamics of Patterns*' and of the NWO-program '*Complexity*', and is member of the program committee of the NWO-Earth and Life Sciences program '*Climate Variability*'. Doelman has (co-)authored more than 50 refereed journal papers; his cumulative number of PhD-students is 18, and his current h-index (Google scholar) is 18.

Prof. dr. U.M. Ebert (url: www.cwi.nl/~ebert/) leads the group MAC3 'Multiscale Modeling and Nonlinear Dynamics' at CWI and is part time professor of physics at TU/e. Having worked on superconductivity, polymer physics, renormalization group theory, nonlinear front dynamics and phytoplankton modeling in Heidelberg, Jerusalem, Essen, Leiden, and Amsterdam, she presently focuses on spark- and lightning-like discharges, on their multiscale modeling as well as on fundamental and technologically driven experiments, and on geophysical phenomena.
Since 1993, she has published 77 articles in international refereed journals and 22 articles in refereed proceedings of international conferences; her H-index is 18 (Google scholar). She has given 25 invited and 5 plenary talks at international conferences. In 2004, she received FOM's Minerva Prize. Her work on lightning below and above thunderclouds is being extensively covered by the media.
She is a member of the steering group of the research school Centrum voor Plasmafysica en Stralingstechnologie, of the program committee of the STW-program on Transient Plasmas and of the FOM-program on Fusion Physics, and of the Koninklijke Hollandsche Maatschappij der Wetenschappen.

Prof. dr. ir. J. Frank (url: www.cwi.nl/˜jason/) is head of the group MAC1 'Dynamical Systems and Numerical Analysis' at CWI (3 permanent staff, 5 PhD students) and adjunct professor of mathematics at the University of Amsterdam in the chair of Numerical Analysis and Dynamical Systems. His research area is the numerical analysis of time-dependent partial differential equations, with an emphasis on structure-preserving methods, long-time integrations, and accuracy of numerical methods for statistical calculations. He works in close contact with scientists from atmospheric and ocean science and recently energy systems. His recent research addresses the equilibrium statistical mechanics of long-time simulations and statistically consistent multiscale closures.

Frank was awarded a Veni grant in 2002 and grants in the NWO/ALW Climate Variability program in 2004 and (with Bokhove, UT) 2005. He serves on the editorial board of SIAM Journal on Scientific Computing, has published 30 research articles, and has an h-index of 10 (Google scholar).

Prof. S.A. van Gils (url: wwwhome.math.utwente.nl/~gilssa/), is head of the group Applied Analysis and Mathematical Physics. Besides interest in environmental water problems, the group focuses on projects in computational neuroscience. In collaboration with Medical Spectrum Twente, a major challenge is to develop cortical models and to study their rhythms and pathologies.
Van Gils is associate editor of the SIAM Journal of Applied Dynamical Systems. He has co-authored 2 books, 24 refereed journal articles and 13 refereed contributions to proceedings; his h-index is 12 (Google scholar). Van Gils is Director of Education of the Mathematics Department, and program leader of the EPAM program on Industrial Mathematics.

Dr. A.J. Homburg (url: www.science.uva.nl/~alejan) is associate professor at the University of Amsterdam, and project leader of the group Applied Analysis and Dynamical Systems.His research interests focus on dynamical systems. He made contributions to various areas in dynamical systems theory, ranging from bifurcation theory, symmetric and reversible differential equations to ergodic and random aspects of dynamical systems. He has supervised four PhD students and published more than 30 journal articles; his h-index is 10 (Google scholar).

Prof. dr. M.A. Peletier (url: www.win.tue.nl/~mpeletie/) is professor of Mathematics at Technische Universiteit Eindhoven, and head of the group Applied Analysis, consisting of nine tenured staff and 14 PhDs and postdocs. His research interests focus on differential equations and their applications to real-world systems. His work spans the spectrum from applied to fundamental, with mathematical contributions to biology, geology, chemistry, pattern formation, solid mechanics, and fluid dynamics, and contributions to the theory of variational calculus and differential equations.
Peletier recieved a VIDI grant in 2003 and became a member of De Jonge Akademie in 2006. He was visiting professor in London (2000), Bath (2007, 2010) and Vancouver (2008), and he serves on the editorial board of three international journals. He has supervised four PhD students and published more than 60 papers, with an h-index of 12 (Google scholar).

Prof. dr. ir. C. Vuik (url: www.ta.twi.tudelft.nl/users/vuik/) is professor of Mathematics at Delft University of Technology and head of the group Numerical Analysis. The group focuses on fast solvers for large linear systems on modern computer architectures and discretization techniques for multiphase flow and other moving boundary problems. Vuik is associate editor of the SIAM Journal on Scientific Computing and editor of the book: Advanced Computational Methods in Science and Engineering, editors: B. Koren and C. Vuik, Springer, Berlin, 2010. In 2009 he was the Burgers Lecturer at the University of Maryland, USA. Furthermore he is the director of the Delft Centre for Computational Science and Engineering. He is one of the six founding members of the 3TU Applied Mathematics Institute (3TU.AMI). Finally he has an h-index of 22 (Google scholar) and published more than 90 papers.

3. Cluster structure

3.1. Organization of the cluster

The cluster will bring together research groups from eight universities and the CWI; the groups and their members are listed below. Each of these groups is associated with several of the research themes outlined above, and there already exist many personal and scientific connections between the groups (see also section C.1.2).

The cluster will be managed by a board, which will initially consist of professors Van den Berg, Broer, Doelman, Frank, and Peletier. The main task of the board will be to coordinate the scientific activities of the cluster regarding the organization of workshops, visitorships, joint research, joint seminars, PhD courses, the initiation of large projects (for instance at the European scale), etc. Moreover, the board will also be active in the (coordination of the) organization of courses at the master level, and in embedding these courses in the 'National Master in Mathematics'-program.

A guiding principle in the allocation of the funds of the cluster is contribution to the long-term national infrastructure in mathematics. In line with this aim, we choose to invest in **tenure-track positions, postdoc and PhD positions, visiting scientists, advisors,** and **workshops.** In addition we willl hold **summer** and **winter schools** for master and beginning PhD students that will ensure the continuity between the various university master tracks and the research of the cluster, and which will bring the students into the community of researchers. We briefly discuss these activities one by one.

- There will be **9 tenure-track positions,** one at each participating institute. These will benefit from NDNS+-funding during four years, after which successful candidates will be adopted by the institutes themselves.
- We plan an internal competition for **6 PhD/postdoc** positions. Emphasis will again be placed on proposals with long-term effects, for instance via other tenure-track positions or incorporation into longer-term activities.
- **Visiting scientists** are essential in bringing new developments abroad to the Dutch mathematical community. We will support a number of visiting scientists each year.
- Personal unions in the form of **advisors** – members of one institute that spend one day a week at another institute – will contribute to the fabric of the community.
- In the past five years the **workshop programme** has led to a large number of workshops in the NDNS+-realm. We will continue and expand this programme, in two ways:
  - by initiating general and focused **NDNS+-workshops**, in promising areas of research, and
  - by financially supporting **other workshops** that are relevant to NDNS+.
- In addition we willl hold **summer** and **winter schools** for master and beginning PhD students that will ensure the continuity between the various university master tracks and the research of the cluster, and which will bring the students into the community of researchers.

Finally, a special type of activity is the **Study Group Mathematics with Industry.** This series of yearly workshops brings together mathematicians of all types with industrialists and other non-mathematicians around specific, practical problems. It is a very successful example of how applied mathematics can be of direct use to the rest of the society, and in addition it provides a hands-on educational experience for master and PhD students and postdocs. The cluster will provide both financial and organisatorial support, and manage the longer-term responsibilities.

The Study Group aims to cover all of mathematics, and in this sense incorporation in a single mathematics cluster is unnatural. However, for reasons of organisational efficiency, and in consort with the other clusters, we propose to centralize the organisation of the Study Groups within a single cluster, under the auspices of the Platform Wiskundig Nederland. Since historically the members of NDNS+ have played a major role in the Dutch Study Groups, NDNS+ is a natural choice.

3.2. Participating Universities

Below we list the partners and the permanent faculty that will be involved with NDNS+, and the themes that they are associated with.

CWI:
- Prof. dr. ir. J. Frank (team leader, Scientific Computing, Stochastic Dynamics)
- Prof. dr. U. Ebert (team leader, Multiple Scales, Patterns and Waves)
- Dr. J.G. Blom (Scientific Computing, Multiple Scales, Stochastic Dynamics)
- Dr. D.T. Crommelin (Stochastic Dynamics, Multiple Scales)
- Prof. dr. W. Hundsdorfer (Scientific Computing)
- Dr. G. Klau (Multiple Scales, Stochastic Dynamics)
- Prof. dr. B. Koren (Scientific Computing)
- Dr. R. Merks (Multiple Scales, Stochastic Dynamics, Scientific Computing)
- Prof. dr. C.W. Oosterlee (Scientific Computing)
- Dr. J. Rademacher (Patterns and Waves)
- Prof. dr. J.H. van Schuppen (Multiple Scales, Stochastic Dynamics)
- Prof. dr. J.G. Verwer (Scientific Computing)

RuG:
- Prof. dr. H.W. Broer (team leader, Bifurcations and Chaos)
- Dr. M.K. Çamlibel (Variational Methods)
- Dr. B. Carpentieri (Scientific Computing)
- Dr. M.E. Dür (Variational Methods)
- Prof. dr. A. van Enter (Stochastic Dynamics)
- Prof. dr. A. van der Schaft (Variational Methods)
- Prof. dr. H.L. Trentelman (Variational Methods)
- Prof. dr. G. Vegter (Bifurcations and Chaos)
- Prof. dr. A. Veldman (Scientific Computing)
- Dr. R.W.C.P. Verstappen (Scientific Computing)
- Prof. dr. H. Waalkens (Bifurcations and Chaos)
- Dr. F.W. Wubs (Scientific Computing)

TUD:
- Prof. dr. ir. C. Vuik (team leader, Scientific Computing, Multiple Scales)
- Prof. dr. ir. A.W. Heemink (Multiple Scales, Stochastic Dynamics)
- Prof. dr. J.M.A.M. van Neerven (Stochastic Dynamics)
- Dr. M.H.A. Haase (Stochastic Dynamics)
- Dr. ir. W.T. van Horssen (Patterns and Waves, Multiple Scales)
- Dr. ir. H.X. Lin (Scientific Computing)
- Dr. H.M. Schuttelaars (Patterns and Waves, Multiple Scales)
- Dr. ir. D.R. van der Heul (Scientific Computing, Multiple Scales)
- Dr. ir. M. van Gijzen (Scientific Computing, Multiple Scales)
- Dr. D.J.P. Lahaye (Scientific Computing, Multiple Scales)
- Dr. ir. J.K. Ryan (Scientific Computing, Multiple Scales)
- Dr. ir. F.J. Vermolen (Scientific Computing, Multiple Scales)

TU/e:
- Prof. dr. M.A. Peletier (team leader, Multiple Scales, Patterns and Waves, Variational Methods, Stochastic Dynamics)
- Dr. M. Anthonissen (Scientific Computing, Multiple Scales)
- Dr. J.H.M. ten Thije Boonkkamp (Scientific Computing, Multiple Scales)
- Prof. dr. E.H. van Brummelen (Scientific Computing, Multiple Scales, Stochastic Dynamics, Variational Methods)
- Dr. M.E. Hochstenbach (Scientific Computing, Multiple Scales)

- Prof. dr. R.M.M. Mattheij (Scientific Computing, Multiple Scales)
- Dr. J.M.L. Maubach (Scientific Computing, Multiple Scales)
- Dr. A. Muntean (Multiple Scales, Patterns and Waves, Stochastic Dynamics)
- Dr. I.S. Pop (Scientific Computing, Multiple Scales, Patterns and Waves)
- Dr. S.W. Rienstra (Patterns and Waves)
- Prof. dr. F. Toschi (Scientific Computing, Multiple Scales, Patterns and Waves)
- Dr. A.S. Tijsseling (Scientific Computing, Multiple Scales, Patterns and Waves)

UvA:
- Dr. A.J. Homburg (team leader, Bifurcations and Chaos, Patterns and waves, Stochastic dynamics)
- Prof. dr. R. Stevenson (Scientific computing, Variational methods, Multiple Scales)
- Dr. J. Brandts (Scientific computing, Variational methods)
- Prof. dr. J.J.O.O. Wiegerinck (Bifurcations and Chaos, Stochastic dynamics)
- Dr. H. Peters (Bifurcations and Chaos, Stochastic dynamics)
- Dr. C. Stolk (Scientific Computing, Multiple Scales, Patterns and waves)

UL:
- Prof. dr. A. Doelman (team leader, Multiple Scales, Patterns and waves)
- Prof. dr. F. den Hollander (Variational methods, Stochastic Dynamics)
- Prof. dr. E. Verbitskiy (Bifurcations and Chaos, Stochastic Dynamics)
- Prof. dr. S. Verduyn Lunel (Patterns and waves, Stochastic Dynamics)
- Dr. O. van Gaans (Variational methods, Stochastic Dynamics)
- Dr. P. Haccou (Patterns and waves, Stochastic Dynamics)
- Dr. S. Hille (Variational methods, Stochastic Dynamics)
- Dr. M.F. De Jeu (Variational methods)
- Dr. V. Rottschäfer (Multiple Scales, Patterns and waves)

UU:
- Prof. dr. O. Diekmann (team leader, Bifurcations and Chaos, Multiple Scales, Patterns and Waves)
- Dr. H. Hanßmann (Bifurcations and Chaos,
- Dr. Y. Kuznetsov (Bifurcations and Chaos, Patterns and Waves)
- Dr. P. Zegeling (Scientific Computing, Multiple Scales)

UT:
- Prof. dr. S.A. van Gils (team leader, Patterns and Waves)
- Dr. O. van Bokhove (Variational Methods, Scientific Computing, Patterns and Waves)
- Dr. M. Botchev (Scientific Computing)
- Dr. R. van Damme (Scientific Computing)
- Prof. dr. B. Geurts (Scientific Computing, Multiple Scales)
- Prof..dr. E. van Groesen (Multiple Scales, Patterns and Waves, Variational Methods)
- Dr. H. Meijer (Bifurcations and Chaos, Patterns and Waves)
- Prof. dr. J.J.W. van de Vegt (Scientific Computing)
- Dr. A. Zagaris (Multiple Scales)

VUA:
- Prof. dr. J.B. van den Berg (team leader, Multiple Scales, Patterns and Waves, Variational Methods, Scientific Computing)
- Prof. dr. J. Hulshof (Multiple Scales, Patterns and Waves)
- Prof. dr. R.C. van der Vorst (Patterns and Waves, Variational Methods)
- Dr. B. Rink (Bifurcations and Chaos, Patterns and Waves, Variational Methods)
- Dr. R. Planqué (Multiple Scales, Variational Methods)
- Dr. F. Pasquotto (Variational Methods)

4. New initiatives

The first five years have created a solid foundation of research infrastructure. The new cluster plans will build on that foundation, further extend it, and branch out into a number of new directions.

New themes:
Two new themes, Variational Methods and Stochastic Dynamics, will reinforce research lines that have grown in recent years, and that we expect to become even more important in the future. When complex systems have a variational structure, methods exploiting this structure may provide strong characterizations of the behaviour of the system. In addition, the global nature of variational methods is an excellent complement to the more local nature of bifurcation theory, and the interaction between these two approaches within the cluster is particularly promising.

The new theme of Stochastic Dynamics reflects the growing importance of stochastic perturbations in deterministic systems, and more generally the increasing interaction between analysis and stochastics.

Large changes have taken place in the theme Scientific Computing. The original conception of this theme was geared towards numerical bifurcation theory. Since the start of the cluster the actual activities have been much broader (see Part B), and the new plans reflect this by extending in the direction of multiscale systems, disparate systems, and the inclusion of stochastic effects, while building on a basis of (adaptive) discretisation and approximation theory. This change also reflects the incorporation of the extensive scientific computing community in the Netherlands.

Finally, the theme Networks and Delays does not reappear in the new plans. While there is scope for a theme connecting networks and discreteness effects with dynamical systems – as e.g. in Systems Biology – the current community in this area in the Netherlands is not very large. However, because of the intrinsic interest of the topic, this theme may reappear in the future.

The role of applications
The orientation of the cluster towards the Earth and Life Sciences has been very fruitful: existing groups have been brought together (e.g. in the case of bio-statistics and analysis), and the existence of the cluster has assisted in creating new, independently funded activities in these directions (e.g. the new Life Sciences group at CWI). Of course the bio-statistics component of NDNS+ joined the STAR cluster when it was created. However, the bridges that have built between the cluster and bio-statistics now have a firm foundation, so that the cluster expects an ongoing interaction with the STAR cluster in the context of the life sciences.

In the new plans we not only retain the application orientation, but we also add a new dimension to it by reinforcing the aspect of engineering. This is reflected in the newly added group at TUD and several of the new members at UT and TU/e, and in the expansion of the theme Scientific Computing.

Education
The new plans also incorporate educational activities for Masters and PhD students. Since the quality of the research at Dutch universities depends so strongly on the quality of the PhD students, it is very much in the interest of Dutch mathematics research to invest in the training and preparation of students, both before and in the PhD period. Taking the format from a successful two-day PhD meeting in 2008, we will organize a multi-day session twice a year. One of these yearly events will be a classical 'Summer school' on a specific topic, while the winter event will focus more on interaction between the Master students and PhDs. Topics for these schools will include Stochastic Differential Equations (most likely in collaboration with STAR), Variational Methods, Computational Bifurcation Theory, Geometric Singular Perturbation Theory, and Lattice Models.

6. Global plan for the investment of funds

The yearly expenses of the cluster plans are detailed below. Amounts are in k€.

Tenure tracks	9x80	720
Postdocs	6x70	420
Workshops		40
Teaching costs		20
Temporary visiting positions		40
Advisors		50
Study Group Math industry		30
Running costs		50
Total		**1370**

Because of the inclusion of all mathematics departments, the budget of NDNS+ rises above the bar of 1M€ set by NWO. We believe that the large size of the cluster justifies this larger expense.

Appendices:
A Indicators EZ
B Indicators OCW

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